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FOUR PHOTON ANNIHILATION

OF AN ELECTRON PAIR

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FOUR PHOTON ANNIHILATION
OF AN ELECTRON PAIR

A dissertation submitted in partial
fulfillment of the requirements
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at

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by

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PREFACE

I am grateful to Dr. James Joseph for suggesting this problem and for the guidance and encouragement which he and Dr. Melvin Ferentz gave me during the course of the investigation.

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INTRODUCTION

This thesis presents a calculation of the differential cross section and a partial cross section in the extreme nonrelativistic limit for the process of four photon in flight annihilation of an electron positron pair. The calculations have been performed in the lowest non-vanishing order of approximation of the Feynman-Dyson formulation of quantum electrodynamics. This is an iterative solution to the S matrix which represents the S matrix as an infinite series in the fine structure constant $\alpha = \frac{e^2}{4\pi} \approx \frac{1}{137}$, and which, for those processes to which it has been applied, has shown excellent agreement with experimental results.

The calculation of the differential cross section is exact and valid for arbitrary energy of the incident particles. There exist earlier calculations of four corner processes such as those of Calogero and Zemach [1960], and Sannikov [1961] but these are restricted to energies in the relativistic region. It is believed that the present work represents the first calculation of a four corner process

at arbitrary energy.

The differential cross section is presented in the form of the reduced cross section, a concept which is developed in Chapter I. This enables one to display the cross section in a relatively abbreviated form while retaining all the information necessary to generate the full differential cross section or any partial cross section. The calculation presented here was feasible at this time only because of this property of the reduced cross section concept.

Although, in principle, the reduced cross section can be employed to obtain any partial cross section, in practice, the difficulties attendant on this determination would be enormous for any but the simplest cases. Consequently the differential cross section has been integrated only for the case, in the extreme nonrelativistic limit, in which two of the photons have energies much smaller than the electron rest energy. Since the cross section for a higher order process of this type will be small it is most likely to be observed in a measurement of the total cross section. However, Joseph [1956] has shown that in any multiple photon emission process the principal contribution to the total cross section occurs when all but two of the photons are soft, i.e., have a collective energy small compared to the electron rest energy.

Another important objective of this research was to explore the possibilities of simplifying the calculation of higher order processes in quantum electrodynamics. As an example of the complexity expected the four photon annihilation process involves twenty-four Feynman diagrams. These would normally require the evaluation of 300 traces which contain on the average 240 terms each. There are available computer programs (Wilcox [1961], Hearn [1964]) to evaluate the individual traces. However the task of combining all the traces and simplifying the resulting expression would have to be completed by hand. Before any progress can be made, therefore, some method must be found to reduce the number of traces which must be calculated.

The concept of the reduced cross section provides one such method. The reduced cross section is defined as the expression obtained from the transition probability by effecting an identity between all terms which differ from one another only by a permutation of identical final state particles. In Chapter I it is shown that by reason of the invariance of the transition probability under any such permutation any partial or differential cross section may be obtained from the reduced cross section.

For the four photon annihilation process it is necessary to evaluate only 17 traces in order to determine

the reduced cross section. Although this represents a very significant reduction from 300 traces there still remains a considerable amount of hand calculation in order to combine the 17 traces and express the reduced cross section in the simplest form.

In a computation of this magnitude the danger of of computational error is always present. In order to minimize the possibility of undetected errors of this type all hand calculations were performed twice. As is well-known by everyone who has had to perform long calculations this double check procedure is not foolproof. A better check is obtained by computing the cross section by an entirely different method if there is one available. In this problem an alternate method is provided by the two component formulation of quantum electrodynamics (Brown [1958], Sannikov [1961]). This formulation permits a direct calculation of the extreme relativistic limit of quantum electrodynamical processes. The computation is simpler than and sufficiently different from the four component computation to provide an independent check in the extreme relativistic limit. For the four photon annihilation process the two component calculation agreed exactly with the extreme relativistic limit of the four component calculation. This agreement, coupled with the double check made on all hand calculations,

provides reasonably complete assurance against the presence of computational error in the reduced cross section for all energies.

The history of the problem of multiple photon production in pair annihilation dates back to 1930 when Dirac considered the two photon annihilation process (Dirac [1930]). The differential cross section for three photon annihilation was not known until 1952 when Mandl and Skyrme [1952] considered the process of double Compton scattering which is related to three photon annihilation through the substitution law.

In 1954 several cases of narrow showers of about twenty high energy photons were observed in cosmic ray events (Schein, Haskin, and Glasser [1954], DeBenedetti, Garelli, Tallone, Vigone, and Wataghin [1954]). In seeking an explanation of Schein's photon shower Gupta [1955] computed the leading term of the cross section for n photon annihilation in the high energy approximation. He made the assumption that all photons in the center of mass system would be emitted in two narrow cones centered on the incident electron and positron momenta and that the largest contribution to the cross section arises when all but two of the photons are soft.

Joseph [1956] proved the validity of Gupta's assumptions and obtained the first four terms in the high energy expansion of the three photon annihilation cross section and the two leading terms in the high energy expansion of the n photon annihilation cross section. Gupta and Joseph both concluded that the Schein photon shower could not be accounted for by electromagnetic processes.

Andreassi, Calucci, Furlan, Peressutti and Cazzola [1962] succeeded in performing a complete integration of the total cross section at arbitrary energy for the three photon annihilation process. They also calculated the first order radiative corrections to the two photon annihilation cross section. Combining these results with the Dirac cross section, i.e., the cross section to lowest order for annihilation into two photons only, they obtained the total cross section to order e^6 for annihilation of an electron positron pair into photons.

In an experiment performed at C.E.R.N., Geneva, Fabiani and his associates (Fabiani, Fidecaro, Finocchiaro, Giacomelli, Harting, Lipman, and Torelli [1962]) measured at energies in the range from 2 to 10 Bev the total cross section for annihilation of an electron positron pair into any number of photons, i.e., their experiment did not distinguish between two photon, three photon, and higher

order annihilation processes. A similar experiment at 800 Mev was performed by Braccini, Ion, Stefanini, Torelli, and Torelli Tosi [1963]. For both experiments the mean values of the results are in surprising agreement with the Dirac cross section, which at first sight tends to cast doubt on the validity of higher order corrections. However, since quantum electrodynamics has been very successful in predicting with a high degree of accuracy the higher order corrections in other processes, such as the magnetic moment of the electron and the Lamb shift, one suspects that in this case the source of the apparent disagreement between experiment and theory is to be sought in experimental uncertainties.

Andreassi et al. discussed the first experiment and concluded that it was not a good measure of the radiative corrections since both the Dirac cross section and their cross section to order e^6 fall within the limits of the experimental error. The quoted range of experimental error for the second experiment is narrower and includes only the Dirac cross section. However this experiment did not directly measure the photon emission annihilation cross section but rather the attenuation of an incident beam of positrons. Because of the numerous corrections which must be included to deduce the annihilation cross section from

the direct experimental results this experiment alone cannot be regarded as a critical test of the validity of the iterative approach to the computation of quantities in quantum electrodynamics.

CHAPTER I

REDUCED CROSS SECTION

In the process of calculating cross sections in quantum electrodynamics, it is found that an extremely large number of terms are generated, many of which combine with one another in the final result. Despite this combination, the final differential cross section for higher order processes involves so many terms (numbered in the thousands) as to make the result almost useless. In this chapter it will be shown that for processes involving groups of identical particles in the final state, a condition which will be realized in all processes of order higher than the third, a considerable reduction of effort can be effected by utilizing the identity of the particles.

Andreassi et al. [1962] utilized an argument of this type in their calculation of three photon pair annihilation. However, whereas their paper implies that the technique is only valid if the total cross section is calculated, it will be shown here that it can be used for any differential or partial cross section.

For any process involving two particles in the initial state, the cross section is given by (Jauch and Rohrlich [1955]):

$$\sigma = (2\pi)^2 \frac{\epsilon_1^2}{F} S_f \bar{S}_i \delta(P_f - P_i) |(f|M|i)|^2 \quad (1)$$

$$F = \sqrt{(p_{1\mu} p_2^\mu)^2 - m_1^2 m_2^2} \quad (2)$$

$p_{1\mu}$ is the energy-momentum four-vector, ϵ_1 , the energy, and m_1 the mass of particle 1. P_i and P_f are respectively the total energy-momentum four-vectors in the initial and final states. The matrix element $(f|M|i)$ is related to the S matrix element by: $(f|S|i) = \delta(P_f - P_i)(f|M|i)$ for an initial state ω_i and a final state $\omega_j \neq \omega_i$. It is a function of the momenta and polarizations or spins of all initial and final state particles.

For simplicity, consider the process with an electron, p , and a positron, q , in the initial state and n photons, k_1, k_2, \dots, k_n , in the final state. Assume further that neither the spin nor the polarization states are observed. Then define the function

$$|(f|M|i)|^2 = M^2(p, q, k_1, \dots, k_n) \quad (3)$$

The prescription according to which the matrix element is calculated insures that the function M^2 is symmetric under an interchange of any two photons, i.e.

$$(i)M^2(p,q,k_1\dots k_n) = M^2(p,q,k_1\dots k_n) \quad (4)$$

where (i) represents any element of the group of permutations of the integers 1 to n .

In order to compute a partial cross section, it is necessary to specify a range of values in momentum phase space for each momentum vector $\underline{k}_1, \dots, \underline{k}_n$. Let $\{x_1\}$ be the set of ordered triplets of numbers which specifies the range of values of \underline{k}_1 . Because of the constraint of energy-momentum conservation, the sets $\{x_1\} \dots \{x_n\}$ are not independent. In general, it is possible to specify only one set, say $\{x_1\}$. The others are then expressed as functions of \underline{k}_1 and of one another depending upon the order of integration chosen.

The result, however, of the functional relations between the \underline{k} 's and the specification of one or possibly more sets $\{x_1\}$ is a region R_1 of momentum phase space defined by the $3n$ dimensional set of points.

$$R_1 = \left\{ \underline{k}_1 = \{x'\}; \underline{k}_2 = \{x''\}; \underline{k}_3 = \{x'''\} \dots \underline{k}_n = \{x^{n'}\} \right\} \quad (5)$$

Every point in the set satisfies both energy-momentum conservation and the conditions we wish to impose on the cross section. In addition if the region R_1 has been properly determined then every physically realizable final state which meets the conditions of the cross section will be included at least once in the set. In practice, the proper determination of R_1 is accomplished by a proper determination of the limits of integration. It often happens that some physically realizable final states are included more than once in the region R_1 . These multiple contributions must be compensated in determining the true cross section, but this can be accomplished by examining the limits of integration for the specific cross section being calculated. In this general discussion we will ignore this effect.

Therefore, the partial cross section for a specified region of final state configurations is

$$\sigma_1 = (2\pi)^2 \frac{\epsilon_1 \epsilon_2}{F} \int_{R_1} d^3k_1 \dots d^3k_n \delta(P_f - P_i) \frac{1}{4} \sum_{\text{spins}} \sum_{\text{pol}} \quad (6)$$

$$M^2(p, q, k_1 \dots k_n) .$$

Now consider a region R_2 of momentum phase space defined by the $3n$ dimensional set of points:

$$R_2 = \left[\underline{k}_2 = \{x'\}; \underline{k}_1 = \{x''\}; \underline{k}_3 = \{x'''\} \dots \underline{k}_n = \{x^{n'}\} \right] \quad (7)$$

Due to the physical indistinguishability of the photons the regions R_1 and R_2 represent the same set of physically realizable final states although they may be distinct regions of momentum phase space. Thus the cross section

$$\sigma_2 = (2\pi)^2 \frac{\epsilon_1 \epsilon_2}{F} \int_{R_2} d^3k_1 \dots d^3k_n \delta(P_f - P_i) \frac{1}{4} \sum_{\text{spins}} \sum_{\text{pol}} M^2(p, q, k_1 \dots k_n) \quad (8)$$

is equal to the cross section σ_1 in Eq. (6).

Now define a region R_1 of phase space by the set of points:

$$R_1 = \left[\underline{k}_\alpha = \{x'\}; \underline{k}_\beta = \{x''\}; \underline{k}_\gamma = \{x'''\} \dots \underline{k}_\nu = \{x^{n'}\} \right] \quad (9)$$

i.e., where the momentum vector \underline{k}_α takes on the set of values $\{x'\}$ which were assigned to \underline{k}_1 in region R_1 , and the sequence of integers $\alpha, \beta, \gamma \dots \nu$ is obtained by

applying the permutation (i) to the sequence 1, 2, 3, ..., n. We will use the notation: $R_i = (i)R_1$ with the understanding that the permutation operator is applied only to the subscripts of the momentum vectors and the permutation (1) is the identity operator. Again the cross section

$$\sigma_i = (2\pi)^2 \frac{\epsilon_1 \epsilon_2}{F} \int_{R_i} d^3 k_1 \dots d^3 k_n \delta(P_f - P_i) \frac{1}{4} \sum_{\text{spins}} \sum_{\text{pol}} M^2(p, q, k_1 \dots k_n) \quad (10)$$

is equal to σ_1 .

Since there are $n!$ elements in the permutation group, the partial cross section can be written

$$\begin{aligned} \sigma &= \frac{1}{n!} \sum_{i=1}^{n!} \sigma_i \\ &= \frac{1}{n!} (2\pi)^2 \frac{\epsilon_1 \epsilon_2}{F} \sum_{i=1}^{n!} \int_{R_i} d^3 k_1 \dots d^3 k_n \delta(P_f - P_i) \frac{1}{4} \sum_{\text{spins}} \sum_{\text{pol}} M^2(p, q, k_1 \dots k_n) \end{aligned} \quad (11)$$

Consider the case in which there exists no overlap between

any of the regions R_i in phase space. Then it is possible to define a composite region

$$R_T = \sum_{i=1}^{n!} R_i \quad (12)$$

and the sum of $n!$ integrals in Eq. (11) can be written as one integral

$$\sigma = \frac{1}{n!} (2\pi)^2 \frac{\epsilon_1 \epsilon_2}{E} \int_{R_T} d^3 k_1 \dots d^3 k_n \delta(P_f - P_i) \frac{1}{4} \sum_{\text{spins}} \sum_{\text{pol}} M^2(p, q, k_1 \dots k_n) \quad (13)$$

The region R_T is a symmetric region of phase space in the sense that any permutation of the momentum vector subscripts maps the region R_T onto itself, i.e., (i) $R_T = R_T$. This invariant property of R_T is clearly a result of its definition and the property of the permutation group:

$$(i) \quad \sum_{j=1}^{n!} (j) = \sum_{j=1}^{n!} (j) \quad (14)$$

Consider the integral

$$I = \int_{R_1} d^3k_1 \dots d^3k_n \delta(P_f - P_i) A(p, q, k_1 \dots k_n) \quad (15)$$

where the function $A(p, q, k_1 \dots k_n)$ is not necessarily invariant under permutations of the subscripts $1, 2 \dots n$.

The value of this integral will not be affected by relabeling the variables of integration provided the relabeling is carried out in both the integrand and the region of integration, thus

$$I = (j)I = \int_{(j)R_1} d^3k_1 \dots d^3k_n \delta(P_f - P_i) (j)A(p, q, k_1 \dots k_n) \quad (16)$$

since the delta function is invariant and the order of integration is immaterial until specific limits of integration are chosen to represent the region. Now suppose there exist in $\sum_{\text{spins}} \sum_{\text{pol}} M^2$, Eq. (13), two terms A and A' which are related by:

$$A'(p, q, k_1, k_2 \dots k_n) = (i)A(p, q, k_1, k_2 \dots k_n)$$

then by the argument just given

$$\begin{aligned}
& \int_{R_T} d^3k_1 \dots d^3k_n \delta(P_f - P_i) \frac{1}{4} A(p, q, k_1 \dots k_n) \\
&= (i) \int_{R_T} d^3k_1 \dots d^3k_n \delta(P_f - P_i) \frac{1}{4} A(p, q, k_1 \dots k_n) \\
&= \int_{R_T} d^3k_1 \dots d^3k_n \delta(P_f - P_i) \frac{1}{4} (i) A(p, q, k_1 \dots k_n) \\
&= \int_{R_T} d^3k_1 \dots d^3k_n \delta(P_f - P_i) \frac{1}{4} A'(p, q, k_1 \dots k_n)
\end{aligned} \tag{17}$$

Thus the terms A and A' give the same contribution to the cross section σ , in the form in which it is given in Eq. (13), and we can replace $A + A'$ by $2A$ in that equation.

In general, there will be many pairs of terms in $\sum_{\text{spins}} \sum_{\text{pol}} M^2$ which differ from one another only by a permutation of the photon momenta subscripts and which consequently give equal contributions to Eq. (13). Define σ_r , the reduced differential cross section, as the function generated from $\frac{1}{4} \sum_{\text{spins}} \sum_{\text{pol}} M^2$ by combining all terms which differ from one another by a permutation of the photon momenta. Then the partial cross section for the process is

given by:

$$\sigma = \frac{1}{n!} (2\pi)^2 \frac{\epsilon_1 \epsilon_2}{F} \int_{R_T} d^3 k_1 \dots d^3 k_n \delta(P_f - P_i) \sigma_r(p, q, k_1 \dots k_n) \quad (18)$$

Note that $\sigma_r(p, q, k_1 \dots k_n)$ is not a symmetric function of the photon momenta.

Recalling the definition of R_T the cross section in Eq. (18) can be written as a sum of $n!$ integrals:

$$\sigma = \frac{1}{n!} (2\pi)^2 \frac{\epsilon_1 \epsilon_2}{F} \sum_{i=1}^{n!} \int_{R_i} d^3 k_1 \dots d^3 k_n \delta(P_f - P_i) \sigma_r(p, q, k_1 \dots k_n) \quad (19)$$

This equation permits us to calculate any partial cross section and any differential cross section, since these are merely partial cross sections with differential ranges of integration, from the reduced differential cross section, σ_r . However, since the regions R_i are distinct one from the other, the application of Eq. (19) requires the determination of $n!$ different sets of integration limits. For some cross sections the following formulation may introduce simplifications.

Consider one of the integrals:

$$I_i = \int_{R_i} d^3k_1 \dots d^3k_n \delta(P_f - P_i) \sigma_r(p, q, k_1 \dots k_n) \quad (20)$$

According to the arguments leading to Eq. (16)

$$\begin{aligned} I_i &= (i)_\ell^{-1} I_i \\ &= \int_{(i)_\ell^{-1} R_i} d^3k_1 \dots d^3k_n \delta(P_f - P_i) (i)_\ell^{-1} \sigma_r(p, q, k_1 \dots k_n) \end{aligned} \quad (21)$$

where $(i)_\ell^{-1}$ is the element of the permutation group which is the left inverse of (i) , i.e., $(i)_\ell^{-1} (i) = 1$. Now

$$(i)_\ell^{-1} R_i = R_1$$

and, therefore,

$$I_i = \int_{R_1} d^3k_1 \dots d^3k_n \delta(P_f - P_i) (i)_\ell^{-1} \sigma_r(p, q, k_1 \dots k_n) \quad (22)$$

After performing similar operations on each integral in Eq. (19) we obtain for the cross section

$$\sigma = \frac{1}{n!} (2\pi)^2 \frac{\epsilon_1 \epsilon_2}{F} \sum_{i=1}^{n!} \int_{R_1} d^3 k_1 \dots d^3 k_n \delta(P_f - P_i) (i)_\ell^{-1} \quad (23)$$

$\sigma_r(p, q, k_1 \dots k_n)$

$$= \frac{1}{n!} (2\pi)^2 \frac{\epsilon_1 \epsilon_2}{F} \int_{R_1} d^3 k_1 \dots d^3 k_n \delta(P_f - P_i) \sum_{i=1}^{n!} (i)_\ell^{-1}$$

$\sigma_r(p, q, k_1 \dots k_n)$

Since the left inverse of any element of the permutation group is unique:

$$\sum_{i=1}^{n!} (i)_\ell^{-1} = \sum_{i=1}^{n!} (i)$$

and

$$\sigma = \frac{1}{n!} (2\pi)^2 \frac{\epsilon_1 \epsilon_2}{F} \int_{R_1} d^3 k_1 \dots d^3 k_n \delta(P_f - P_i) \sum_{i=1}^{n!} (i) \quad (24)$$

$\sigma_r(p, q, k_1 \dots k_n)$

Equations (19) and (24) were derived on the assumption of no overlap between the regions R_i . This

restriction can be removed by requiring that in Eq. (13) the integration over the overlapping regions be repeated as many times as there is overlap. This insures that the integral over R_T in Eq. (13) is still equivalent to the sum of integrals over the R_1 's in Eq. (11).

The existence of an overlap implies that in the region R_1 some physically realizable final states have been counted more than once. Consequently the formula for the partial cross section given in Eqs. (6-24) must be modified to compensate for these multiple contributions. As mentioned above this must be accomplished by examining the limits of integration for each individual case.

The assumption made at the beginning of this chapter that the process under consideration was an n photon pair annihilation was unnecessary. There is nothing in the proof of Eq. (24) which limits its validity to any specific processes. If there are n electrons rather than n photons in the final state then the matrix element $\langle f|M|i \rangle$ is antisymmetric under an interchange of any two electrons. However M^2 defined in Eq. (3) will still be symmetric so that Eq. (4) and the derivation of Eq. (24) are still valid. In the general case of n photons, m electrons and r positrons in the final state the operation

$\frac{1}{n!} \dots \sum_{i=1}^{n!} (i) \sigma_r$ in Eq. (24) must be replaced by $\frac{1}{n!} \frac{1}{m!}$

$\frac{1}{r!} \dots \sum_{i=1}^{n!} \sum_{j=1}^{m!} \sum_{k=1}^{r!} (i)(j)(k) \sigma_r$. In this new expression

(i), (j), and (k) are photon, electron, and positron permutation operators respectively and σ_r is obtained from the complete differential cross section by three successive steps of combining terms which are equivalent under photon, electron and positron permutations.

CHAPTER II

FOUR QUANTUM ANNIHILATION IN FLIGHT

The reduced differential cross section is calculated to the lowest order, nonvanishing term of the S matrix expansion for the process of in-flight annihilation of an electron-positron pair into four photons.

The basic Feynman diagram for this process is shown in Fig. 1, where p is the electron four momentum, q is the positron four momentum and k_i is the four momentum of the i 'th photon. In order to compute the S matrix element for this process it is necessary to consider Fig. 1 and the twenty-three additional Feynman

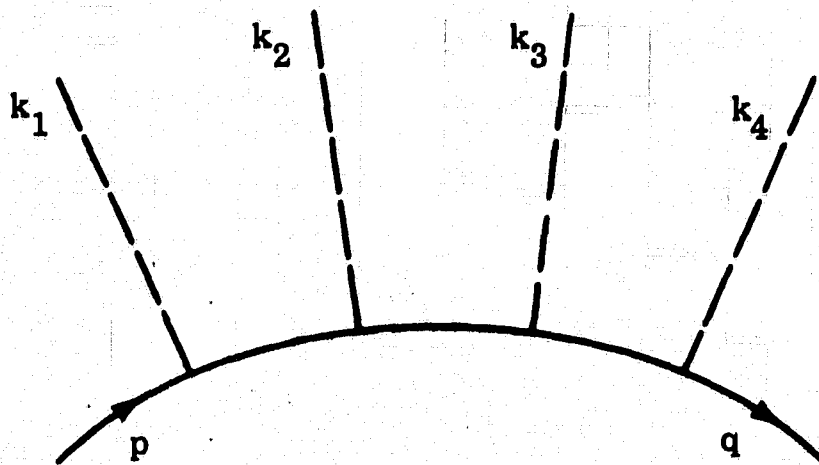


Fig. 1 Basic Feynman Diagram

diagrams which arise from it by making all possible rearrangements of the order in which the four photons are emitted. For convenience these permutations are listed in Table I of the Appendix. The meaning of the notation used for the permutation group elements in column 2 is that the permutation (i,j,k) operating on an arbitrary sequence of the four photons has the effect of replacing photon i by photon j , photon j by photon k , and photon k by photon i .

A product of two or more permutations is to be read from right to left so that the permutation $(1,2)(3,2)$ operating on the sequence $[1\ 2\ 3\ 4]$ has the following effect:

$$(1,2)(3,2) [1\ 2\ 3\ 4] = (1,2) [1\ 3\ 2\ 4] = [2\ 3\ 1\ 4]$$

In the following the permutations will be referred to by the symbols in the first column of Table I. They form a complete group and some useful properties of the group are shown in Table II of the Appendix.

The S matrix element for the diagram in Fig. 1 is given according to the usual rules (Jauch and Rohrlich [195]) by:

$$\begin{aligned}
(f|S|i)_1 &= i^3 (2\pi)^{-5} \frac{m e^4}{4} \sqrt{\frac{1}{\epsilon_1 \epsilon_2 \omega_1 \omega_2 \omega_3 \omega_4}} \\
&> \int \int \int d^4 p_1 d^4 p_2 d^4 p_3 \bar{v}(q) \gamma^\mu \delta(p_3 - k_4 + q) \\
&> e_\mu(k_4) \frac{i \not{p}_3 - m}{p_3^2 + m^2} \gamma^\nu \delta(p_2 - k_3 - p_3) e_\nu(k_3) \\
&> \frac{i \not{p}_2 - m}{p_2^2 + m^2} \gamma^\lambda \delta(p_1 - k_2 - p_2) e_\lambda(k_2) \frac{i \not{p}_1 - m}{p_1^2 + m^2} \gamma^\rho \\
&> \delta(p - k_1 - p_1) e_\rho(k_1) u(p) ,
\end{aligned} \tag{25}$$

where m and e are the mass and charge of the electron, ϵ_1 , ϵ_2 and ω_1 are the energies of the electron, positron, and i 'th photon, and \bar{v} and u are the spinors corresponding to an ingoing positron and electron, respectively. The metric employed is $g_{11} = g_{22} = g_{33} = 1 = -g_{00}$, and the summation convention for repeated Greek indices is employed. γ^μ is the usual 4×4 gamma matrix, $e_\mu(k_i)$ is the polarization four vector of the i 'th photon and the notation \not{p} represents $p_\mu \gamma^\mu$. The system of units for which $\hbar = c = 1$ is chosen. The integrations are performed easily.

$$\begin{aligned}
(f|S|i)_1 &= i^3 (2\pi)^{-5} \frac{m e^4}{4 \sqrt{\epsilon_1 \epsilon_2 \omega_1 \omega_2 \omega_3 \omega_4}} \bar{v}(q) \not{\epsilon}(k_4) \\
&>< \frac{i(k_4 - q) - m}{(k_4 - q)^2 + m^2} \not{\epsilon}(k_3) \frac{i(p - k_1 - k_2) - m}{(p - k_1 - k_2)^2 + m^2} \not{\epsilon}(k_2) \quad (26) \\
&>< \frac{i(p - k_1) - m}{(p - k_1)^2 + m^2} \not{\epsilon}(k_1) u(p) \delta(p + q - k_1 - k_2 - k_3 - k_4)
\end{aligned}$$

Introduce the notation $(f|M|i)$ such that

$$(f|S|i)_1 = \delta(p + q - k_1 - k_2 - k_3 - k_4) (f|M|i)_1$$

where the subscript 1 indicates that this is that part of the matrix element corresponding to Feynman diagram 1, that is Fig. 1.

The total S matrix element is a sum of the matrix elements for each of twenty-four diagrams,

$$(f|S|i) = \sum_{j=1}^{24} (f|S|i)_j \quad (27)$$

However

$$(f|S|i)_j = (j) (f|S|i)_1 ,$$

this is the matrix element corresponding to diagram j can be obtained by applying permutation j to the matrix element corresponding to diagram 1.

For the cross section it is necessary to evaluate the quantity

$$\begin{aligned}
 |(f|M|i)|^2 &= \left| \sum_{j=1}^{24} (f|M|i)_j \right|^2 \\
 &= \sum_{j=1}^{24} \sum_{k=1}^{24} (f|M|i)_j (f|M|i)_k^* \\
 &= \sum_j \sum_k \left\{ (j)(f|M|i)_1 \right\} \left\{ (k)(f|M|i)_1^* \right\} \\
 &= \sum_j (j) \sum_k \left\{ (f|M|i)_1 (j)^{-1}(k)(f|M|i)_1^* \right\}
 \end{aligned} \tag{28}$$

where $(j)^{-1}$ is the permutation group element which is right inverse to the element (j) . Now $(j)^{-1}(k)$ is another element of the group and, for a fixed value of j , it will, according to Eq. (14), generate each element of the group when k is varied from 1 to 24. Therefore, the sum can be replaced by:

$$|(f|M|i)|^2 = \sum_j (j) \sum_k \left\{ (f|M|i)_1(k) (f|M|i)_1^* \right\} \quad (29)$$

If the spin and polarization states are not observed, the quantity required for the cross section is:

$$\begin{aligned} & \frac{1}{4} \sum_{\text{spins}} \sum_{\text{pol}} |(f|M|i)|^2 \\ &= \sum_j (j) \sum_k \frac{1}{4} \sum_{\text{spins}} \sum_{\text{pol}} \left\{ (f|M|i)_1(k) \right. \\ & \qquad \qquad \qquad \left. (f|M|i)_1^* \right\} \end{aligned} \quad (30)$$

According to the discussion in Chapter I, the reduced differential cross section is

$$\sigma_r = 24 \sum_k \frac{1}{4} \sum_{\text{spins}} \sum_{\text{pol}} \left\{ (f|M|i)_1(k) (f|M|i)_1^* \right\} \quad (31)$$

This expression is considerably smaller than the full differential cross section. Further reductions are possible, however, by considering the explicit form of the expressions $\left\{ (f|M|i)_1(k) (f|M|i)_1^* \right\}$.

These are obtained from Eq. (26) and Table I, recalling that $k_1^2 = 0$ and $p^2 = q^2 = -m^2$.

$$\begin{aligned}
 (f|M|i)_1 (f|M|i)_1^* &= (2\pi)^{-10} \frac{m^2 e^8}{16 \epsilon_1 \epsilon_2 \omega_1 \omega_2 \omega_3 \omega_4} \bar{v}(q) \not{\epsilon}(k_4) \\
 &>< \frac{i(k_4 - q) - m}{[-2q \cdot k_4]} \not{\epsilon}(k_3) \frac{i(p - k_1 - k_2) - m}{[-2p \cdot k_1 - 2p \cdot k_2 + 2k_1 \cdot k_2]} \not{\epsilon}(k_2) \\
 &>< \frac{i(p - k_1) - m}{[-2p \cdot k_1]} \not{\epsilon}(k_1) u(p) \bar{u}(p) \not{\epsilon}(k_1) \frac{i(p - k_1) - m}{[-2p \cdot k_1]} \\
 &>< \not{\epsilon}(k_2) \frac{i(p - k_1 - k_2) - m}{[-2p \cdot k_1 - 2p \cdot k_2 + 2k_1 \cdot k_2]} \not{\epsilon}(k_3) \frac{i(k_4 - q) - m}{[-2q \cdot k_4]} \\
 &>< \not{\epsilon}(k_4) v(q). \tag{32}
 \end{aligned}$$

$$\begin{aligned}
 (f|M|i)_1 (f|M|i)_2^* &= (2\pi)^{-10} \frac{m^2 e^8}{16 \epsilon_1 \epsilon_2 \omega_1 \omega_2 \omega_3 \omega_4} \bar{v}(q) \not{\epsilon}(k_4) \\
 &>< \frac{i(k_4 - q) - m}{[-2q \cdot k_4]} \not{\epsilon}(k_3) \frac{i(p - k_1 - k_2) - m}{[-2p \cdot k_1 - 2p \cdot k_2 + 2k_1 \cdot k_2]} \not{\epsilon}(k_2) \\
 &>< \frac{i(p - k_1) - m}{[-2p \cdot k_1]} \not{\epsilon}(k_1) u(p) \bar{u}(p) \not{\epsilon}(k_2) \frac{i(p - k_2) - m}{[-2p \cdot k_2]}
 \end{aligned}$$

$$\begin{aligned}
&>< \not{\epsilon}(\underline{k}_1) \frac{i(\not{p}-\not{k}_1-\not{k}_2) - m}{[-2p.k_1-2p.k_2+2k_1.k_2]} \not{\epsilon}(\underline{k}_3) \frac{i(\not{k}_4-\not{q}) - m}{[-2q.k_4]} \\
&>< \not{\epsilon}(\underline{k}_4) v(q) .
\end{aligned} \tag{33}$$

Since the other expressions are similar they will not be written out. After performing the spin and polarization summations they can be written in the form:

$$\frac{1}{4} \sum_{\text{spins}} \sum_{\text{pol}} (f|M|i)_1 (f|M|i)_k^* = A \frac{T_k}{D_k} \tag{34}$$

$$A = (2\pi)^{-10} \frac{m^2 e^8}{16 \epsilon_1 \epsilon_2 \omega_1 \omega_2 \omega_3 \omega_4}$$

The quantities T_k and D_k are listed in Table III of the Appendix where the notation

$$[ij] = p.k_i + p.k_j - k_i.k_j$$

and

$$m = -im_1$$

has been introduced for convenience.

The reduced differential cross section now takes the form

$$\sigma_r = 24 A \sum_{k=1}^{24} \frac{T_k}{D_k} . \quad (35)$$

Examination of the expressions $\frac{T_k}{D_k}$ in Table III reveals that some can be written as permutations of others. Consider the effect of applying the permutation (1,3,2) to $\frac{T_8}{D_8}$

$$(1,3,2) \frac{T_8}{D_8} = \left\{ 2^6 (q \cdot k_4)^2 p \cdot k_3 p \cdot k_1 [13][12] \right\}^{-1} \frac{1}{4m^2} \frac{1}{4} \text{Tr} \left\{ \gamma^\mu \right. \\ \left. [k_4 - q + m_1] \gamma^\nu [p - k_3 - k_1 + m_1] \gamma^\lambda [p - k_3 + m_1] \gamma^\rho [p + m_1] \gamma_\lambda \right. \\ \left. [p - k_1 + m_1] \gamma_\nu [p - k_1 - k_2 + m_1] \gamma_\rho [k_4 - q + m_1] \gamma_\mu [q - m_1] \right\} \quad (36)$$

Since traces of gamma matrices are invariant under both cyclic interchange of factors and reversal of the order of factors we can rewrite this as

$$(1,3,2) \frac{T_8}{D_8} = \left\{ 2^6 (q \cdot k_4)^2 p \cdot k_3 p \cdot k_1 [13][12] \right\}^{-1} \frac{1}{4m^2} \frac{1}{4}$$

$$\text{Tr} \left\{ \gamma_\mu [k_4 - q + m_1] \right.$$

$$\gamma_\rho [p - k_1 - k_2 + m_1] \gamma_\nu [p - k_1 + m_1] \gamma_\lambda [p + m_1] \gamma^\rho [p - k_3 + m_1] \gamma^\lambda$$

$$\left. [p - k_3 - k_1 + m_1] \gamma^\nu [k_4 - q + m_1] \gamma^\mu [q - m_1] \right\} \quad (37)$$

This, however, is equal to $\frac{T_9}{D_9}$ since the indices on the gamma matrices are dummy and can be interchanged freely as long as the relative positions of two matrices with the same index are not altered.

An examination of Table II indicates that

$$(8) (9) = (9) (8) = 1$$

and that similar relations hold for six other pairs of permutations, namely (10), (11); (12), (13); (14), (15); (16), (21); (17), (19); (18), (20). This suggests that the expressions $\frac{T_k}{D_k}$ corresponding to these six pairs be examined for similar behavior. In analogy with the (8), (9) pair the first member of each pair is to be operated upon by the permutation corresponding to the second member.

$$(1,4,2) \frac{T_{10}}{D_{10}} = \left\{ 2^6 q \cdot k_4 q \cdot k_2 p \cdot k_4 p \cdot k_1 [14][12] \right\}^{-1} \frac{1}{4m^2} \frac{1}{4} \text{Tr} \left\{ \gamma^\mu \right.$$

$$[k_2 - \not{q} + m_1] \gamma^\nu [\not{p} - k_4 - k_1 + m_1] \gamma^\lambda [\not{p} - k_4 + m_1] \gamma^\rho [\not{p} + m_1]$$

$$\gamma_\lambda [\not{p} - k_1 + m_1] \gamma_\mu [\not{p} - k_1 - k_2 + m_1] \gamma_\nu [k_4 - \not{q} + m_1] \gamma_\rho [\not{q} - m_1] \left. \right\}$$

Rewriting this in reverse order:

$$(1,4,2) \frac{T_{10}}{D_{10}} = \frac{T_{11}}{D_{11}} \quad (38)$$

$$(1,4,3) \frac{T_{12}}{D_{12}} = \left\{ 2^6 q \cdot k_4 q \cdot k_3 p \cdot k_4 p \cdot k_1 [24][12] \right\}^{-1} \frac{1}{4m^2} \frac{1}{4} \text{Tr} \left\{ \gamma^\mu \right.$$

$$[k_3 - \not{q} + m_1] \gamma^\nu [\not{p} - k_4 - k_2 + m_1] \gamma^\lambda [\not{p} - k_4 + m_1] \gamma^\rho$$

$$[\not{p} + m_1] \gamma_\nu [\not{p} - k_1 + m_1] \gamma_\lambda [\not{p} - k_1 - k_2 + m_1] \gamma_\mu$$

$$[k_4 - \not{q} + m_1] \gamma_\rho [\not{q} - m_1] \left. \right\}$$

$$(1,4,3) \frac{T_{12}}{D_{12}} = \frac{T_{13}}{D_{13}} \quad (39)$$

$$(2,4,3) \frac{T_{14}}{D_{14}} = \left\{ 2^6 q \cdot k_4 q \cdot k_3 (p \cdot k_1)^2 [14][12] \right\}^{-1} \frac{1}{4m^2} \frac{1}{4}$$

$$\text{Tr} \left\{ \gamma^\mu [k_3 - \not{q} + m_1] \gamma^\nu [\not{p} - k_1 - k_4 + m_1] \gamma^\lambda [\not{p} - k_1 + m_1] \right.$$

$$\gamma^\rho [\not{p} + m_1] \gamma_\rho [\not{p} - k_1 + m_1] \gamma_\nu [\not{p} - k_1 - k_2 + m_1] \gamma_\mu$$

$$\left. [k_4 - \not{q} + m_1] \gamma_\lambda [\not{q} - m_1] \right\}$$

$$(2,4,3) \frac{T_{14}}{D_{14}} = \frac{T_{15}}{D_{15}} \quad (40)$$

$$(1,4,3,2) \frac{T_{16}}{D_{16}} = \left\{ 2^6 q \cdot k_4 q \cdot k_3 p \cdot k_4 p \cdot k_1 [14][12] \right\}^{-1} \frac{1}{4m^2} \frac{1}{4}$$

$$\text{Tr} \left\{ \gamma^\mu [k_3 - \not{q} + m_1] \gamma^\nu [\not{p} - k_4 - k_1 + m_1] \gamma^\lambda [\not{p} - k_4 + m_1] \right.$$

$$\gamma^\rho [\not{p} + m_1] \gamma_\lambda [\not{p} - k_1 + m_1] \gamma_\nu [\not{p} - k_1 - k_2 + m_1]$$

$$\left. \gamma_\mu [k_4 - \not{q} + m_1] \gamma_\rho [\not{q} - m_1] \right\}$$

$$(1,4,3,2) \frac{T_{16}}{D_{16}} = \frac{T_{21}}{D_{21}} \quad (41)$$

$$(1,3,4,2) \frac{T_{17}}{D_{17}} = \left\{ 2^6 q \cdot k_4 q \cdot k_2 p \cdot k_3 p \cdot k_1 [13][12] \right\}^{-1} \frac{1}{4m^2} \frac{1}{4}$$

$$\text{Tr} \left\{ \gamma^\mu [k_2 - \not{d} + m_1] \gamma^\nu [\not{p} - k_3 - k_1 + m_1] \gamma^\lambda \right.$$

$$[\not{p} - k_3 + m_1] \gamma^\rho [\not{p} + m_1] \gamma_\lambda [\not{p} - k_1 + m_1] \gamma_\mu$$

$$\left. [\not{p} - k_1 - k_2 + m_1] \gamma_\rho [k_4 - \not{d} + m_1] \gamma_\nu [\not{d} - m_1] \right\}$$

$$(1, 3, 4, 2) \frac{T_{17}}{D_{17}} = \frac{T_{19}}{D_{19}} \quad (42)$$

$$(1, 4, 2, 3) \frac{T_{18}}{D_{18}} = \left\{ 2^6 q \cdot k_4 q \cdot k_2 p \cdot k_4 p \cdot k_1 [34][12] \right\}^{-1} \frac{1}{4m^2} \frac{1}{4}$$

$$\text{Tr} \left\{ \gamma^\mu [k_2 - \not{d} + m_1] \gamma^\nu [\not{p} - k_4 - k_3 + m_1] \gamma^\lambda \right.$$

$$[\not{p} - k_4 + m_1] \gamma^\rho [\not{p} + m_1] \gamma_\nu [\not{p} - k_1 + m_1] \gamma_\mu$$

$$\left. [\not{p} - k_1 - k_2 + m_1] \gamma_\lambda [k_4 - \not{d} + m_1] \gamma_\rho [\not{d} - m_1] \right\}$$

$$(1, 4, 2, 3) \frac{T_{18}}{D_{18}} = \frac{T_{20}}{D_{20}} \quad (43)$$

These seven relations permit us to write for the reduced cross section:

$$\sigma_r = 24 A \left\{ \sum_{k=1}^7 \frac{T_k}{D_k} + 2 \frac{T_8}{D_8} + 2 \frac{T_{10}}{D_{10}} + 2 \frac{T_{12}}{D_{12}} \right\}$$

$$\begin{aligned}
& + 2 \frac{T_{14}}{D_{14}} + 2 \frac{T_{16}}{D_{16}} + 2 \frac{T_{17}}{D_{17}} + 2 \frac{T_{18}}{D_{18}} \\
& + \frac{T_{22}}{D_{22}} + \frac{T_{23}}{D_{23}} + \frac{T_{24}}{D_{24}} \} \quad (44)
\end{aligned}$$

This is the simplest form in which the reduced cross section can be written without actually computing the traces, T_i . After explicit expressions for the traces have been computed further simplification can be attained by detailed comparisons of the individual terms of each trace.

The traces in Eq. (44) were computed on an IBM 7090 computer using the program written by Wilcox [1961]. In order to check the program, T_1 was calculated completely by hand; T_2 , T_7 , and T_{22} were calculated by hand assuming $p = q$, $k_1 = k_2 = k_3 = k_4$; T_6 assuming $k_1 = k_2 = k_3 = k_4 = 0$; and T_{12} assuming $p = q$, $k_1 = k_2 = k_3 = k_4 = 0$. In all cases, the machine results agreed with the results computed by hand.

The total number of terms in all seventeen traces exceeded 4000. However, this number was reduced considerably and the form of the remaining terms simplified by: 1) combining those terms which differ only by a permutation operation, 2) rearranging and combining terms so as to eliminate one or both of the factors $[ij]$ in the

denominators, 3) reducing the number of terms by the application of energy-momentum conservation relations. Because of the extremely large number of terms involved it was easier to apply this reduction process directly on the expression $\frac{T_i}{D_i}$ before combining all 17 expressions, and further because some of the traces are reasonably short the entire set was divided into two groups which were treated separately.

The group $\frac{T_i}{D_i}$ where $i = 1, 2, 3, 5, 6, 7, 22$ were treated in the following manner. Each expression was surveyed systematically to determine if any terms in the expression can arise by permutations on any other terms in the same expression. As an example of this process consider $i = 2$. D_2 is invariant under the permutation (1,2) so that if two terms in T_2 differ only by this permutation these terms can be combined in $\frac{T_2}{D_2}$.

Next, terms of the form $k_i \cdot k_m$ were replaced by expressions involving the trinomials which appear in the denominator. These expressions were determined by application of energy momentum conservation. For $i = 3$ the following relations were obtained

$$k_1 \cdot k_2 = - [12] + p \cdot k_1 + p \cdot k_2$$

$$k_2 \cdot k_3 = - [23] + p \cdot k_2 + p \cdot k_3$$

$$k_1 \cdot k_4 = - [23] + q \cdot k_1 + q \cdot k_4$$

$$k_1 \cdot k_3 = [12] + [23] - p \cdot k_2 - q \cdot k_4$$

$$k_2 \cdot k_4 = [12] + [23] + p \cdot k_4 + q \cdot k_2 - p \cdot q + m^2$$

The motive for this was that terms like $k_2 \cdot k_3$, since they involve the angle between two photon momenta, lead to more complicated integrations than terms like $p \cdot k_1$ which involve only one angle with respect to the coordinate system defined in terms of the initial momenta.

Finally, the entire expression was surveyed for the existence of groups of terms which by application of energy momentum conservation can be rewritten in such a way that factors involving photon momenta, hence integration variables, are replaced by the factors $p \cdot q$ and m^2 . Throughout all phases of this process full use was made of the fact that the denominator of any given expression $\frac{T_i}{D_i}$ is invariant under some permutation so that this permutation can be applied indiscriminately to terms in T_i .

The expressions $\frac{T_i}{D_i}$ for $i = 4, 8, 10, 12, 14,$

16, 17, 18, 23, 24 were treated in basically the same manner. However, these expressions are so long that an attempt was made to reduce the work by searching first for terms common to more than one expression. This search was conducted directly on the computer print outs and considerably reduced the total number of terms which had to be treated by the aforementioned process. The method of search can be demonstrated by considering $\frac{T_8}{D_8}$ and $\frac{T_{16}}{D_{16}}$. When $q.k_4$ is removed from D_8 and $q.k_1$ is removed from D_{16} the remaining expressions are equal. Consequently if there exists a term $q.k_4A$, where A represents the product of three other scalar products, in T_8 and a term $q.k_1A$ in T_{16} then these terms would be equal in the reduced cross section. Similarly when $q.k_4p.k_1$ is removed from D_8 and $q.k_1p.k_1$ is removed from D_{16} the remaining expressions are equal and invariant under the permutation (1,3). Consequently, if $q.k_4p.k_1A$ exists in T_8 and $q.k_1p.k_1B$ exists in T_{16} then these terms would be equivalent if either $A = B$ or $A = (1,3)B$.

In carrying out this comparison process it was essential to devise a bookkeeping method which was both simple and secure in the sense of positively accounting for all changes. The method employed was suggested by the format of the computer print out - five terms in each horizontal

row. On blank sheets of paper a grid of boxes was set up with five boxes in each row and 24 rows on each sheet. The rows on the computer print out for each T_i were numbered consecutively from 1 and a sufficient number of sheets were identified with that print out and the rows on these sheets numbered accordingly. Thus each term on the computer print out was uniquely identified with one box in the grid. The convention was established that blank boxes represent the number 1. As comparisons between traces were made a cumulative record was kept in each box of the number of times that the corresponding term in the corresponding print out had occurred. Whenever two equivalent terms were found the term was removed from the grid with the lower T_i index and added to the grid with the higher index. Thus, if the term $32 q.k_4A$ appeared in T_8 and $-64q.k_1A$ in T_{16} and if the corresponding grid boxes were both empty then a zero was entered in the T_8 grid box and a $1/2$ in the T_{16} grid box. If the T_8 box already contained a zero this indicated that the term had previously been transferred elsewhere and so no longer existed in T_8 . In general, terms from grids with low indices were transferred to grids with high indices, i.e., $T_8 \rightarrow T_{16}$, $T_{10} \rightarrow T_{17}$, etc. On the right hand side of each grid a record was kept of which comparisons had been made.

The expressions T_{18} , T_{23} , and T_{24} were not compared with the remaining seven expressions because they each contain $[12][34]$ in the denominator and no permutation will transform this into one of the other denominators. It must be recalled that, in Eq. (44), $\frac{T_i}{D_i}$ for $i = 8, 10, 12, 14, 16, 17, 18$ appears doubled. Consequently, when a term from T_4 was transferred to any of the grids with these indices it was multiplied by a factor $1/2$ to compensate for the subsequent doubling. Similarly, when a term from T_{18} was transferred to the T_{23} or T_{24} grids it was doubled.

After all possible comparisons between T_i 's were completed they were each separately treated in the manner described above for the shorter expressions. Finally all $\frac{T_i}{D_i}$ were combined according to the prescription given by Eq. (44) and terms which differed by a permutation operation were combined. The result, σ_r , is given in Table IV of the Appendix. There are slightly more than 1000 terms in this expression which indicates the considerable reduction of terms in the reduced cross section as compared with the full differential cross section.

In an attempt to eliminate errors all calculations, with the exception of the computer runs, described in this and in subsequent chapters were performed twice.

CHAPTER III

EXTREME RELATIVISTIC LIMIT

In the limit of very high electron energies, $\beta \approx 1$, the Dirac equation can be written as two uncoupled equations each of which describes a particle with a 2-component wave function (Rose [1961]). A method for calculating the cross sections for processes involving high energy electrons by means of this 2-component formulation has been described by Brown [1958] and Sannikov [1961]. As a further check on the accuracy of the calculations described in Chapter II, the 4 photon annihilation cross section was computed again using the 2-component formulation.

The procedure is analogous to that described above, the major difference arising from the use of two dimensional Pauli spin matrices in place of the four dimensional gamma matrices. In place of the four vector γ_μ whose components are the gamma matrices, the four vectors $\vec{\sigma}_\mu^+$ and $\vec{\sigma}_\mu^-$ must be used. These are defined by:

$$\vec{\sigma}_\mu^+ = (\underline{\sigma}, 1) \quad ; \quad \vec{\sigma}_\mu^- = (\underline{\sigma}, -1) \quad (45)$$

where \underline{g} is the three vector composed of the three Pauli spin matrices and i represents the unit matrix multiplied by the imaginary quantity, $\sqrt{-1}$. We are using Sannikov's notation. The metric is $g_{11} = g_{22} = g_{33} = g_{00} = 1$, and the fourth component of physical four vectors is imaginary.

By utilizing the properties of the Pauli spin matrices (Rose [1961]) the quantities $\overset{+}{\sigma}_{\mu}$ and $\bar{\sigma}_{\mu}$ can be shown to satisfy the commutation relations,

$$\overset{+}{\sigma}_{\mu} \bar{\sigma}_{\nu} + \bar{\sigma}_{\nu} \overset{+}{\sigma}_{\mu} = 2 \delta_{\mu\nu} \quad (46)$$

$$\bar{\sigma}_{\mu} \overset{+}{\sigma}_{\nu} + \bar{\sigma}_{\nu} \overset{+}{\sigma}_{\mu} = 2 \delta_{\mu\nu}$$

the completeness condition

$$\sum_{\mu} (\overset{+}{\sigma}_{\mu})^{\alpha\beta} (\bar{\sigma}_{\mu})_{\rho\sigma} = 2 \delta_{\sigma}^{\alpha} \delta_{\rho}^{\beta}, \quad (47)$$

and the relations

$$\begin{aligned} \epsilon \overset{+}{\sigma}_{\mu} \epsilon &= \widetilde{\bar{\sigma}_{\mu}} \\ \epsilon \bar{\sigma}_{\mu} \epsilon &= \widetilde{\overset{+}{\sigma}_{\mu}} \end{aligned} \quad (48)$$

$\tilde{\sigma}$ is the transposed matrix and ϵ is defined in terms of the antisymmetric Pauli matrix, $\epsilon = i\sigma_2$. Equations (48) follow directly from the commutation relations and the fact that σ_2 is the only antisymmetric Pauli matrix.

We modify the procedure of Chapter II by writing, in place of Eq. (28),

$$\begin{aligned}
 |(f|M|i)|^2 &= \left| \sum_{j=1}^{24} (f|M|i)_j \right|^2 \\
 &= \sum_{j=1}^{24} \sum_{k=1}^{24} (f|M|i)_j (f|M|i)_k^* \\
 &= \frac{1}{2} \sum_{j=1}^{24} \sum_{k=1}^{24} \left\{ (f|M|i)_j (f|M|i)_k^* + (f|M|i)_k \right. \\
 &\quad \left. (f|M|i)_j^* \right\} \\
 &= \frac{1}{2} \sum_{j=1}^{24} \sum_{k=1}^{24} (j) \left\{ (f|M|i)_1 (j)^{-1} (f|M|i)_k^* \right. \\
 &\quad \left. + (j)^{-1} (f|M|i)_k (f|M|i)_1^* \right\}
 \end{aligned}$$

$$| (f|M|i) |^2 = \frac{1}{2} \sum_{j=1}^{24} (j) \sum_{k=1}^{24} \left\{ (f|M|i)_1 (f|M|i)_k^* + (f|M|i)_k (f|M|i)_1^* \right\} \quad (49)$$

In the last step we have again used the fact that, for fixed j , $(j)^{-1}(k)$ generates the entire permutation group when k is varied from 1 to n . Then the reduced cross section is

$$\sigma_r = 12 \sum_{k=1}^{24} \frac{1}{4} \sum_{\text{spins}} \sum_{\text{pol}} \left\{ (f|M|i)_1 (f|M|i)_k^* + (f|M|i)_k (f|M|i)_1^* \right\} \quad (50)$$

In order that Eq. (50) represent the extreme relativistic limit of the process calculated in Chapter II it is necessary to divide by 4 in determining the average over initial spin states even though the annihilation of a left-handed electron with a right-handed positron and vice versa are strictly forbidden in the 2 component formulation.

Now when $(j) = (k)^{-1}$, that is the right inverse of (k) ,

$$\begin{aligned}
& (f|M|i)_1 (f|M|i)_k^* + (f|M|i)_k (f|M|i)_1^* \\
&= (k) \left\{ (f|M|i)_j (f|M|i)_1^* + (f|M|i)_1 (f|M|i)_j^* \right\} \\
&= (k) \left\{ (f|M|i)_1 (f|M|i)_j^* + (f|M|i)_j (f|M|i)_1^* \right\} \quad (51)
\end{aligned}$$

Therefore:

$$\begin{aligned}
\sigma_r = 12 \left\{ \sum_k + 2 \sum_k \right\} \frac{1}{4} \sum_{\text{spins}} \sum_{\text{pol}} \left\{ (f|M|i)_1 (f|M|i)_k^* \right. \\
\left. + (f|M|i)_k (f|M|i)_1^* \right\} \quad (52)
\end{aligned}$$

where in the \sum_k sum k takes the values 1-7, 22-24,

and in the \sum_k sum k takes the values 8, 10, 12,

14, 16, 17, 18.

For any value of k , $\sum_{\text{spins}} \sum_{\text{pol}} (f|M|i)_1 (f|M|i)_k^*$

is an expression of the form

$$\sum_{\text{spins}} \sum_{\text{pol}} (f|M|i)_1 (f|M|i)_k^* = B \text{Tr} \left\{ \bar{\sigma}_\mu \hat{a}_1 \bar{\sigma}_\nu \hat{a}_2 \bar{\sigma}_\lambda \hat{a}_3 \right. \quad (53)$$

$$\left. \bar{\sigma}_\rho \hat{a}_4 \bar{\sigma}^\mu \hat{a}_5 \bar{\sigma}^\nu \hat{a}_6 \bar{\sigma}^\lambda \hat{a}_7 \bar{\sigma}^\rho \hat{a}_8 \right\}$$

In this expression $\hat{a}_1 = (a_1)_\alpha \bar{\sigma}^\alpha$, where a_1 is any four vector and B is real. The order of the superscripts μ, ν, λ, ρ in the second half of the expression is arbitrary. As it stands Eq. (53) is difficult to calculate. However, since $(f|M|i)_k$ is a one dimensional matrix product $(f|M|i)_k^* = (f|M|i)_k^\dagger$, where the dagger indicates the Hermitian adjoint. Therefore,

$$\begin{aligned} & \sum_{\text{spins}} \sum_{\text{pol}} \left[(f|M|i)_1 (f|M|i)_k^* + (f|M|i)_k (f|M|i)_1^* \right] \\ &= \sum_{\text{spins}} \sum_{\text{pol}} \left[(f|M|i)_1 (f|M|i)_k^\dagger + \left\{ (f|M|i)_1 (f|M|i)_k^\dagger \right\}^\dagger \right] \\ &= B \text{Tr} \mathfrak{U} + \left\{ B \text{Tr} \mathfrak{U} \right\}^\dagger \\ &= B \left\{ \text{Tr} \mathfrak{U} + \left\{ \text{Tr} \mathfrak{U} \right\}^* \right\} \quad (54) \end{aligned}$$

Now it can be shown, by using Eqs. (46), (47), and (48), that

$$\begin{aligned}
 & \text{Tr} [\bar{a}_1 \bar{a}_2 \bar{a}_3 \dots \bar{a}_{2n}] + \left\{ \text{Tr} [\bar{a}_1 \bar{a}_2 \bar{a}_3 \dots \bar{a}_{2n}] \right\}^* \\
 &= \text{Tr} [\bar{a}_1 \bar{a}_2 \bar{a}_3 \dots \bar{a}_{2n} + \bar{a}_{2n} \dots \bar{a}_3 \bar{a}_2 \bar{a}_1] \\
 &= \text{Tr} [\bar{a}_1 \bar{a}_2 \bar{a}_3 \dots \bar{a}_{2n}] \quad (55)
 \end{aligned}$$

where the last trace is to be computed on the assumption that $\bar{a}_1, \bar{a}_2, \dots$ are scalar products of the four momenta with four dimensional gamma matrices. It is important to notice that Eq. (55) is valid only if: 1) the left hand member does not explicitly contain any $\bar{\sigma}_\mu$ or $\bar{\sigma}_\mu$ matrices, 2) the number of factors is even, and 3) the super signs alternate. The importance of this relation is that by repeated application of the commutation and completeness relations any trace of the type in Eq. (53) can be reduced to a sum of traces satisfying the conditions for validity of Eq. (55).

Thus, the reduced cross section, Eq. (52), can be written as a sum of quantities which are formally equivalent to the traces which arise in the four component formulation. Consequently, the computer program which Wilcox wrote only

for the four component formulation can also be used in the two component. The reduced cross section for four photon annihilation of an electron-positron pair was calculated in this manner and the results agreed exactly with the extreme relativistic limit of the results of Chapter II. This agreement considerably increases our confidence in the validity of the results of Chapter II for arbitrary energy.

CHAPTER IV

NONRELATIVISTIC APPROXIMATION

The reduced cross section has been integrated for the special case in the nonrelativistic limit in which two photons have energies small compared to the electron rest energy and one hard photon is emitted in a small cone in the forward direction. This special case has been chosen to simplify both the limits of integration and the subsequent integrations. It is of value since it has been shown by Joseph [1956] that in a multiple photon process the principal contributions to the cross section arise when all but two of the photons are soft. A group of photons whose total energy is small compared to the electron rest energy are called soft.

Let us choose photons k_1 and k_2 to be soft and let photon k_3 be emitted in a narrow cone centered on the initial positron direction \underline{q} . In the laboratory system the nonrelativistic limit, $\beta \rightarrow 0$, of the reduced cross section is obtained by making the following substitutions:

$$\begin{aligned}
p &\rightarrow 0, & q &\rightarrow 0 \\
\epsilon_1 &\rightarrow m, & \epsilon_2 &\rightarrow m \\
p \cdot k_i &= p \cdot \underline{k}_i - \epsilon_1 \omega_i \rightarrow -m\omega_i \\
q \cdot k_i &= q \cdot \underline{k}_i - \epsilon_2 \omega_i \rightarrow -m\omega_i \\
p \cdot q &= p \cdot q - \epsilon_1 \epsilon_2 \rightarrow -m^2 \\
[ij] &= p \cdot k_i + p \cdot k_j - k_i \cdot k_j \\
&\rightarrow -m\omega_i - m\omega_j - k_i \cdot k_j
\end{aligned} \tag{56}$$

When i, j, ℓ, m represent four distinct photons

$$[ij] = p \cdot k_i + p \cdot k_j - k_i \cdot k_j = q \cdot k_\ell + q \cdot k_m - k_\ell \cdot k_m \tag{57}$$

but in the nonrelativistic limit

$$q \cdot k_\ell + q \cdot k_m - k_\ell \cdot k_m \rightarrow -m\omega_\ell - m\omega_m - k_\ell \cdot k_m \tag{58}$$

Therefore, in this limit,

$$[ij] = [\ell m] \tag{59}$$

i.e., $[12] = [34]$, $[13] = [24]$ etc. Using these approximations the reduced cross section in the nonrelativistic

limit is

$$\sigma_r = 24 A \kappa \quad (60)$$

and is given in Table V of the Appendix. Notice that A still contains a factor $\epsilon_1^{-1} \epsilon_2^{-1}$ needed to cancel the factor $\epsilon_1 \epsilon_2$ in Eq. (24) so that the only effect of the small electron-positron relative velocity is contained in F .

The limits of integration for ω_1 and ω_2 are

$$\Delta_1^m \leq \omega_1 \leq \Delta_2^m \quad (61)$$

$$\Delta_1^m \leq \omega_2 \leq \Delta_2^m$$

where $\Delta_1 < \Delta_2 \ll 1$. The relations for conservation of energy and momentum

$$\begin{aligned} \omega_1 + \omega_2 + \omega_3 + \omega_4 &= 2m \\ \underline{k}_1 + \underline{k}_2 + \underline{k}_3 + \underline{k}_4 &= 0 \end{aligned} \quad (62)$$

can be represented by Fig. 2. For given values of \underline{k}_1 and \underline{k}_2 the point A moves on an ellipsoid whose foci are at points P and Q and which satisfies the relation:

$$\omega_3 + \omega_4 = 2m - \omega_1 - \omega_2$$

It can be seen from the figure that the smallest value of ω_3 or ω_4 is x . Now ω_3 and ω_4 must both be larger than $\Delta_1 m$ which is the experimental energy resolution. However, they must also be larger than $\Delta_2 m$ in order that only two photons are soft. Therefore the range of values of \underline{k}_1 and \underline{k}_2 must be restricted so that $x > \Delta_2 m$ for all points in the range. The value of x for arbitrary \underline{k}_1 and \underline{k}_2 can be determined from the figure when \underline{k}_3 and \underline{k}_4 both lie along the line KK' .

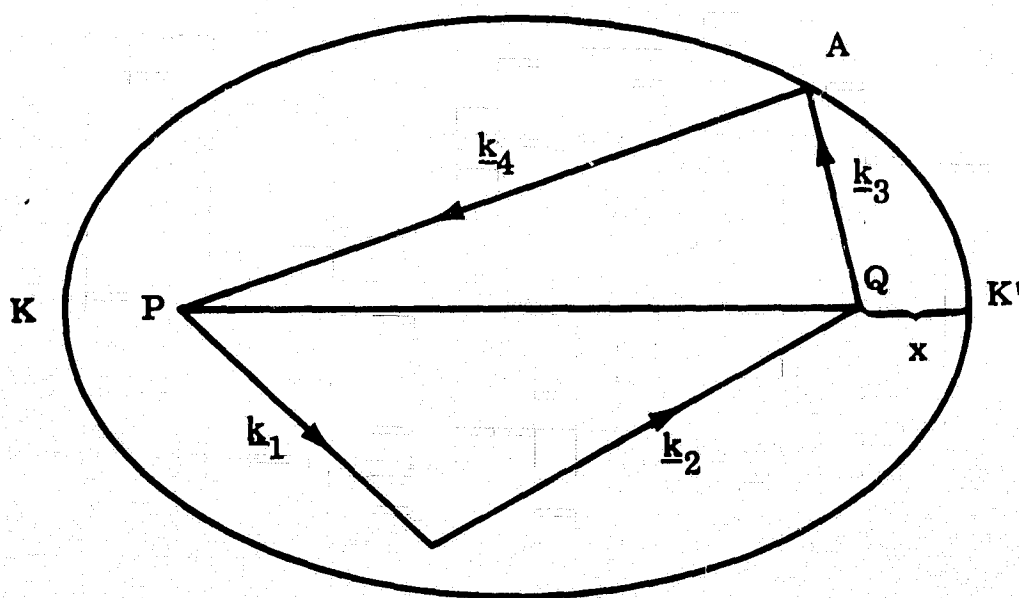


Fig. 2 Conservation of Energy and Momentum

$$\begin{aligned}
 x &= \frac{1}{2} \left\{ \omega_3 + \omega_4 - |PQ| \right\} \\
 &= \frac{1}{2} \left\{ 2m - \omega_1 - \omega_2 - |PQ| \right\}
 \end{aligned}
 \tag{63}$$

The largest value of $|PQ|$ occurs when \underline{k}_1 and \underline{k}_2 are parallel and $\omega_1 = \omega_2 = \Delta_2 m$. Therefore, the smallest value of x for any orientation of \underline{k}_1 and \underline{k}_2 subject to the conditions of Eq. (61) is

$$\begin{aligned}
 x_{\min} &= \frac{1}{2} \left\{ 2m - 4\Delta_2 m \right\} \\
 x_{\min} &= m - 2\Delta_2 m
 \end{aligned}
 \tag{64}$$

Thus if we wish only two photon energies to be less than $\Delta_2 m$ it is necessary to impose the condition

$$\Delta_2 < \frac{1}{3}
 \tag{65}$$

Conversely, the largest value of either ω_3 or ω_4 can be seen to be

$$\omega = \omega_3 + \omega_4 - x$$

$$\begin{aligned}
 &= 2m - \omega_1 - \omega_2 - x \\
 &= m - \frac{\omega_1 + \omega_2}{2} + \frac{1}{2} |PQ|
 \end{aligned} \tag{66}$$

so that, for given values of ω_1 and ω_2 , ω is largest when $|PQ|$ is largest which occurs when \underline{k}_1 and \underline{k}_2 are parallel

$$\omega_{\max} = m \tag{67}$$

Therefore, in the region of integration

$$\Delta_1 m \leq \omega_1 \leq \Delta_2 m$$

$$\Delta_1 m \leq \omega_2 \leq \Delta_2 m$$

$$0 \leq \theta_1, \theta_2 \leq \pi$$

$$0 \leq \theta_3 \leq \delta$$

$$0 \leq \varphi_1, \varphi_2, \varphi_3 \leq 2\pi$$

$$\Delta_1 < \Delta_2 < \frac{1}{3}$$

(68)

the energies ω_3 and ω_4 lie between m and $m - 2\Delta_2 m$, and \underline{k}_4 lies within a cone centered on the negative polar axis.

It should be noticed that Eqs. (68), which constitute the region R_1 of Eq. (24), include all physical states twice due to the indistinguishability of photons 1 and 2. Thus there will exist overlap between any two regions R_i and R_j and an additional factor $\frac{1}{2}$ must be inserted in the expression for the cross section. In order to avoid further overlap caused by an intersection between the cones defined by the allowed direction of \underline{k}_3 and \underline{k}_4 we must impose the condition

$$\cos \theta \geq \frac{\Delta_2}{1 - \Delta_2} \quad (69)$$

which can be seen by the following argument.

$$\hat{x} \cdot \underline{k}_4 = - \hat{x} \cdot \underline{k}_1 - \hat{x} \cdot \underline{k}_2 - \hat{x} \cdot \underline{k}_3$$

where \hat{x} is a unit vector in the direction of the polar axis. Now $\frac{1}{\omega_4} \hat{x} \cdot \underline{k}_4$ and $\frac{1}{\omega_3} \hat{x} \cdot \underline{k}_3$ are the direction cosines of \underline{k}_4 and \underline{k}_3 respectively so that

$$\frac{1}{\omega_4} \hat{x} \cdot \underline{k}_4 \leq \frac{1}{\omega_3} \hat{x} \cdot \underline{k}_3 \quad (70)$$

must be satisfied for all values of \underline{k}_4 and \underline{k}_3 in order that the cones shall not intersect. Since

$$\omega_4 = 2m - \omega_1 - \omega_2 - \omega_3$$

$$-\hat{x} \cdot \underline{k}_1 - \hat{x} \cdot \underline{k}_2 - \hat{x} \cdot \underline{k}_3 \leq \frac{1}{\omega_3} \hat{x} \cdot \underline{k}_3 [2m - \omega_1 - \omega_2 - \omega_3]$$

$$\text{or} \quad \cos \theta_3 \geq \frac{-\hat{x} \cdot \underline{k}_1 - \hat{x} \cdot \underline{k}_2}{2m - \omega_1 - \omega_2} \quad (71)$$

Since $\Delta_2 < \frac{1}{3}$ the largest value of the right hand side can be written

$$\frac{-\hat{x} \cdot \underline{k}_1 - \hat{x} \cdot \underline{k}_2}{2m - \omega_1 - \omega_2} \leq \frac{\omega_1 + \omega_2}{2m - \omega_1 - \omega_2} \leq \frac{\Delta_2}{1 - \Delta_2} \quad (72)$$

Therefore Eq. (71) will always be satisfied if δ is chosen to satisfy Eq. (69).

Consequently, the cross section for 2 soft photons is

$$\sigma_{2S} = \frac{1}{2} \frac{1}{24} (2\pi)^2 \frac{\epsilon_1 \epsilon_2}{F} \int_{R_1} d^3 k_1 d^3 k_2 d^3 k_3 d^3 k_4 \delta(P_f - P_i)$$

$$\sum_{i=1}^{24} (i) \sigma_r \quad (73)$$

The sum over the permutations can be written

$$\begin{aligned} \sum_{i=1}^{24} (i) &= \left\{ 1 + (1,2) \right\} \left\{ 1 + (3,4) \right\} \left\{ 1 + (1,3) + (1,4) + (2,3) \right. \\ &\quad \left. + (2,4) + (1,3) (2,4) \right\} \\ &\equiv \left\{ 1 + (1,2) \right\} G \end{aligned} \quad (74)$$

But, since R_1 is symmetric under the permutation $(1,2)$,

$$\begin{aligned} &\int_{R_1} d^3k_1 d^3k_2 d^3k_3 d^3k_4 \delta(P_f - P_i) G \sigma_r \\ &= \int_{R_1} d^3k_1 d^3k_2 d^3k_3 d^3k_4 \delta(P_f - P_i) (1,2) G \sigma_r \end{aligned} \quad (75)$$

and

$$\sigma_{2S} = \frac{1}{24} (2\pi)^2 \frac{\epsilon_1 \epsilon_2}{F} \int_{R_1} d^3k_1 d^3k_2 d^3k_3 d^3k_4 \delta(P_f - P_i) G \sigma_r \quad (76)$$

The integration is performed first over d^3k_4 and then over $d\omega_3$

$$\sigma_{2S} = \frac{1}{24} (2\pi)^2 \frac{\epsilon_1 \epsilon_2}{F} \int_{R_1} d^3k_1 d^3k_2 d\omega_3 \omega_3^2 \left| \frac{\partial(2m - \omega_1 - \omega_2 - \omega_3 - \omega_4)}{\partial \omega_3} \right|^{-1} G \sigma_r \quad (77)$$

The integrand in Eq. (77) is to be evaluated at

$$\begin{aligned} \underline{k}_1 + \underline{k}_2 + \underline{k}_3 + \underline{k}_4 &= 0 \\ \omega_1 + \omega_2 + \omega_3 + \omega_4 &= 2m \end{aligned} \quad (78)$$

after performing the operation G on σ_r . Since the limits on θ_3 and φ_3 are independent of the other variables \underline{k}_3 may be chosen as the polar axis of the coordinate system for \underline{k}_1 and \underline{k}_2 . From the form of σ_r it can be seen that θ_{12} , θ_{13} , and θ_{23} , where θ_{ij} is the angle between \underline{k}_i and \underline{k}_j , are the only angles which will appear in the final integrand. Therefore, the final integration over θ_3 and φ_3 will be trivial. This is a consequence of the spherical symmetry which has been introduced by the approximation

$p = q = 0$. Defining

$$E_1 = 2m - \omega_1 - \omega_2 \quad (79)$$

$$Q = -\underline{k}_1 - \underline{k}_2$$

we solve Eqs. (78) for ω_3 and ω_4

$$\omega_3 = \frac{E_1^2 - Q^2}{2(E_1 - \hat{k}_3 \cdot Q)} \quad (80)$$

$$\omega_4 = \frac{E_1^2 - 2E_1 \hat{k}_3 \cdot Q + Q^2}{2(E_1 - \hat{k}_3 \cdot Q)}$$

Then, since $E_1 = \omega_3 + \omega_4$ and $Q = \underline{k}_3 + \underline{k}_4$ so that $|Q| \leq E_1$,

$$\left| \frac{\partial(2m - \omega_1 - \omega_2 - \omega_3 - \omega_4)}{\partial \omega_3} \right| = \frac{E_1 - \hat{k}_3 \cdot Q}{\omega_4} \quad (81)$$

and

$$\sigma_{2S} = \frac{1}{24} (2\pi)^2 \frac{\epsilon_1 \epsilon_2}{F} \int_{R_1} d^3 k_1 d^3 k_2 d\Omega_3 \omega_3 \omega_4 \frac{E_1^2 - Q^2}{2(E_1 - \hat{k}_3 \cdot Q)^2} G \sigma_r \quad (82)$$

Choosing \underline{k}_3 as polar axis and setting $\theta_{13} = \theta_1$, $\theta_{23} = \theta_2$

$$\sigma_{2S} = \frac{1}{24} (2\pi)^2 \frac{\epsilon_1 \epsilon_2}{F} \int_{R_1} d^3k_1 d^3k_2 d\Omega_3 \omega_3 \omega_4$$

$$>< \frac{2m^2 - 2m(\omega_1 + \omega_2) + \omega_1 \omega_2 (1 - \cos\theta_{12})}{[2m - \omega_1(1 - \cos\theta_1) - \omega_2(1 - \cos\theta_2)]^2} G \sigma_r . \quad (83)$$

The factor $\omega_3 \omega_4$ has been left in the integrand to cancel a similar factor which appears in the denominator of σ_r .

The reduced cross section, σ_r , in the nonrelativistic limit contains approximately 150 terms so that $G \sigma_r$ will contain about 1800 terms. Some of these, of course, can be combined but the large majority of them will not combine. Note that in Eq. (83) it is no longer possible to combine terms which are permutations of one another. This is only permissible with a symmetric region of integration. In order to integrate Eq. (83), we make use of the fact that $\frac{\omega_1}{m}$ and $\frac{\omega_2}{m}$ are quantities small compared to unity throughout the region of integration. We expand the integrand of Eq. (83) in powers of $\frac{\omega_1}{m}$ and $\frac{\omega_2}{m}$ and retain

all terms of order $\left(\frac{\omega_1}{m}\right)^{-6}$, $\left(\frac{\omega_1}{m}\right)^{-5}$, and $\left(\frac{\omega_1}{m}\right)^{-4}$, where ω_i represents both ω_1 and ω_2 , that is the term $\left(\frac{\omega_1}{m}\right)^{-3} \left(\frac{\omega_2}{m}\right)^{-3}$ is of order $\left(\frac{\omega_1}{m}\right)^{-6}$ and $\left(\frac{\omega_1}{m}\right)^{-3} \left(\frac{\omega_2}{m}\right)^{-2}$ is of order $\left(\frac{\omega_1}{m}\right)^{-5}$. There are no terms of order $\left(\frac{\omega_1}{m}\right)^{-7}$ or lower in the integrand of Eq. (83). As an example of this expansion, the factor multiplying $G \sigma_r$ becomes:

$$\frac{2m^2 - 2m(\omega_1 + \omega_2) + \omega_1\omega_2(1 - \cos\theta_{12})}{[2m - \omega_1(1 - \cos\theta_1) - \omega_2(1 - \cos\theta_2)]^2} \quad (84)$$

$$= \frac{1}{2} \left\{ 1 - \frac{H}{m} - \frac{1}{4} \frac{B^2}{m^2} - \frac{1}{2} \frac{BH}{m^2} + \frac{3}{4} \frac{H^2}{m^2} + \frac{K^2}{2m^2} \right\}$$

$$B = \omega_1 + \omega_2$$

$$K^2 = \omega_1\omega_2(1 - \cos\theta_{12})$$

$$H = \omega_1 \cos\theta_1 + \omega_2 \cos\theta_2$$

It is necessary to carry the expansion to the order indicated since there exist in $G \sigma_r$ terms of order $\left(\frac{\omega_1}{m}\right)^{-6}$.

Since there are two variables ω_1 and ω_2 in the

expansion it is necessary to impose restrictions on Δ_1 and Δ_2 to insure that all terms of order $(\frac{\omega_1}{m})^{-\ell}$ shall be smaller than all terms of order $(\frac{\omega_1}{m})^{-\ell-1}$ throughout the region of integration. This condition will be fulfilled if we define

$$\Delta_2 = x\Delta_1$$

and require

$$\Delta_2^2 < \Delta_1 < \Delta_2 \quad (85)$$

$$x^2 \Delta_1 < 1 < x$$

or

$$1 < x < \frac{1}{\sqrt{\Delta_1}}$$

After expanding the integrand of Eq. (83), the integration is simple and the cross section becomes:

$$\begin{aligned} \sigma_{2S} = \frac{1}{3F} e^8 (2\pi)^{-5} (1 - \cos\theta) \left\{ \Delta_2^2 \ln \frac{2\Delta_2}{\Delta_1 + \Delta_2} \right. \\ \left. - \Delta_1^2 \ln \frac{\Delta_1 + \Delta_2}{2\Delta_1} - \frac{1}{2} (\Delta_2 - \Delta_1)^2 \right\} \quad (86) \end{aligned}$$

In the laboratory frame of reference, $p = 0$, $\epsilon_1 = m$ and in the nonrelativistic limit, $|q| = m\beta$, $\epsilon_2 = m\sqrt{1 + \beta^2}$ so that

$$F = m^2 \beta$$

and

$$\sigma_{2S} = \frac{2^3}{3\pi} \frac{r_o^2 \alpha^2}{\beta} (1 - \cos\delta) \quad (87)$$

$$> \times \left\{ \Delta_2^2 \ln \frac{2\Delta_2}{\Delta_1 + \Delta_2} - \Delta_1^2 \ln \frac{\Delta_1 + \Delta_2}{2\Delta_1} - \frac{1}{2} (\Delta_2 - \Delta_1)^2 \right\}$$

In this equation r_o is the classical electron radius $\frac{e^2}{4\pi m}$, and α is the fine structure constant $\frac{e^2}{4\pi}$.

Equation (87) has been derived on the assumption that δ satisfies Eq. (69). However, the cross section for $\delta = \pi$ can be obtained very simply. When no restrictions are imposed on δ then the cross section in Eq. (82) must be

written as a sum of two integrals:
$$\left[\int_{R_1'} \dots + \frac{1}{2} \int_{R_1''} \dots \right].$$

The region R_1' is that portion of R_1 no part of which is invariant under the permutation (3,4) while R_1'' is that

portion of R_1 all of which is invariant under (3,4). When δ satisfies Eq. (69) R_1' vanishes and Eq. (87) is the correct expression for the cross section. When δ does not satisfy Eq. (69) but is less than π then both integrals must be evaluated. When $\delta = \pi$ then R_1' vanishes and the cross section can be obtained from Eq. (87) by the substitution $\delta = \pi$ and multiplication by $1/2$.

It is now possible to estimate the total cross section in the nonrelativistic limit. In order to obtain the principal contribution we should choose the largest value for both Δ_2 and $x = \frac{\Delta_2}{\Delta_1}$ consistent with the approximations which have been made. Since Δ_2 is essentially the ratio of successive orders in the expansion in $\frac{\omega_1}{m}$ it is reasonable to assign the maximum value, $\frac{1}{3}$, to Δ_2 . It is also reasonable to assign the maximum value, $\Delta_1^{-1/2}$, to x since this insures that all neglected terms of the integrand are smaller than all included terms throughout the range of integration. With these values the total cross section in the nonrelativistic limit is approximately: $\frac{2^4}{3\pi}$

$\frac{r_o^2 \alpha^2}{\beta} (1.2 \times 10^{-2})$. This cross section is related to the lifetime for four photon annihilation of the singlet, ground state ($1^1 S_0$) of positronium. Following the calculation in Jauch and Rohrlich [1955] the lifetime (τ)

is given by:

$$\frac{1}{\tau} = 4 \rho \beta \sigma$$

where ρ is the density of electrons and the factor 4 is included because the in-flight annihilation cross section was averaged over four possible initial states but selection rules prohibit four photon positronium annihilation except from the singlet state. For the 1^1S_0 state

$$\rho = \frac{\alpha^3 m^3}{8\pi}$$

and the lifetime is

$$\tau \approx \frac{3\pi^2}{2^3} \cdot \frac{1}{\alpha^7 m} \cdot \frac{10^2}{1.2}$$

$$\approx 3.6 \times 10^{-4} \text{ sec.}$$

CONCLUSION

A technique has been developed which simplifies cross section computations in quantum electrodynamics. It involves the determination of a reduced cross section from which any differential or partial cross section can be calculated. The reduced cross section for the four photon annihilation of an electron positron pair has been found and it has been integrated for a special case in the non-relativistic limit. On the basis of this the total cross section for free positron electron annihilation in the non-relativistic limit has been estimated to be $\frac{2^4}{3\pi} \frac{r_o^2 \alpha^2}{\beta} (1.2 \times 10^{-2})$. The lifetime for four photon annihilation of the $1 \text{ } ^1S_0$ state of positronium has been estimated as 3.6×10^{-4} sec.

In addition to these quantitative results this thesis represents an exploration of the feasibility of simplifying calculations in quantum electrodynamics. The economy in computation offered by the reduced cross section concept is very real. A casual glance at Eq. (24) might lead to the conclusion that nothing has been gained since

the operation $\sum_{i=1}^{n!} (i)$ on σ_r will reverse the process by which the reduced cross section has been obtained. Such a conclusion is invalid for two reasons. First the substantial reduction in the number of traces which must be calculated and manipulated represents a definite economy in time and reduces the possibility of error. Secondly there exist many partial cross sections for which the region of definition in phase space contains overlap. In such cases the permutation operator under which the region of integration is invariant can be removed from the sum as in Eqs. (74-76). The extreme example of this is, of course, the total cross section for

which $\sum_{i=1}^{n!} (i) \sigma_r$ reduces to $n! \sigma_r$.

Despite these advantages the reduced cross section technique will not, unfortunately, solve the computational problem presented by high order processes. It simply does not reduce the labor sufficiently. The process of four photon annihilation is just barely manageable. In order to determine the differential cross section for five photon annihilation it would be necessary to evaluate 7,260 traces! This number can be reduced to 73 by utilizing the reduced cross section. More important, however, than the number of

traces is the length of each trace. On the basis of the rule that the largest possible number of terms in the trace of a product of n matrices is $(n-1)!!$ (Jauch and Rohrlich [1955]), the longest trace for the five photon case can contain up to 300 times the number of terms as the longest trace for four photons. The traces which are needed in a cross section never approach the very large number of terms predicted by this rule. However, even if the increase is only a factor of ten, which is probably not an overestimate, one feels fairly safe in predicting that the five photon annihilation process will never be evaluated by the methods used in this thesis. It may be possible with computer programs which will both calculate the traces and determine the reduced cross section although it is questionable whether the resulting expression, because of its great length, could be made the basis of useful calculations.

Meanwhile further work is required on the four photon annihilation process. An attempt should be made to calculate the total cross section in the extreme relativistic limit in order to compare it with the results of Budini and Furlan [1963] who have estimated the upper limit on the cross section for n photon annihilation at high energies. In addition there are other four corner processes such as pair production in electron-electron scattering which may be

amenable to solution by the method of the reduced cross section.

APPENDIX

TABULAR MATERIAL FOR THE FOUR PHOTON ANNIHILATION CROSS SECTION

In order not to interrupt the continuity of the text, all tabular material is presented here.

TABLE I
PERMUTATIONS OF FOUR PHOTONS

Symbol	Permutation Group Element	Effect on Photon Sequence 1 2 3 4
(1)	(1) (2) (3) (4)	1 2 3 4
(2)	(1,2) (3) (4)	2 1 3 4
(3)	(1,3) (2) (4)	3 2 1 4
(4)	(1,4) (2) (3)	4 2 3 1
(5)	(2,3) (1) (4)	1 3 2 4
(6)	(2,4) (1) (3)	1 4 3 2
(7)	(3,4) (1) (2)	1 2 4 3
(8)	(1,2,3) (4)	2 3 1 4
(9)	(1,3,2) (4)	3 1 2 4
(10)	(1,2,4) (3)	2 4 3 1
(11)	(1,4,2) (3)	4 1 3 2
(12)	(1,3,4) (2)	3 2 4 1
(13)	(1,4,3) (2)	4 2 1 3
(14)	(2,3,4) (1)	1 3 4 2
(15)	(2,4,3) (1)	1 4 2 3
(16)	(1,2,3,4)	2 3 4 1
(17)	(1,2,4,3)	2 4 1 3
(18)	(1,3,2,4)	3 4 2 1
(19)	(1,3,4,2)	3 1 4 2
(20)	(1,4,2,3)	4 3 1 2
(21)	(1,4,3,2)	4 1 2 3
(22)	(1,2) (3,4)	2 1 4 3
(23)	(1,3) (2,4)	3 4 1 2
(24)	(1,4) (2,3)	4 3 2 1

TABLE II

PROPERTIES OF THE PERMUTATION GROUP

$(i) (i) = (1)$	for	$i = 1 \text{ to } 7, 22 \text{ to } 24.$
$(i) (i+1) = (1)$	for	$i = 8, 10, 12, 14$
$(i+1) (i) = (1)$	for	$i = 8, 10, 12, 14$
$(1\ 6) (2\ 1) = (2\ 1) (1\ 6) = 1$		
$(1\ 7) (1\ 9) = (1\ 9) (1\ 7) = 1$		
$(1\ 8) (2\ 0) = (2\ 0) (1\ 8) = 1$		

TABLE III
THE QUANTITIES D_k AND T_k IN

$$\frac{1}{4} \sum_{\text{spins}} \sum_{\text{pol}} (f | M | i)_1 (f | M | i)_k^* = A \frac{T_k}{D_k}$$

$$D_1 = 2^6 (q.k_4)^2 (p.k_1)^2 [1\ 2]^2$$

$$D_2 = 2^6 (q.k_4)^2 p.k_1 p.k_2 [1\ 2]^2$$

$$D_3 = 2^6 (q.k_4)^2 p.k_1 p.k_3 [1\ 2] [2\ 3]$$

$$D_4 = 2^6 q.k_1 q.k_4 p.k_1 p.k_4 [1\ 2] [2\ 4]$$

$$D_5 = 2^6 (q.k_4)^2 (p.k_1)^2 [1\ 2] [1\ 3]$$

$$D_6 = 2^6 q.k_2 q.k_4 (p.k_1)^2 [1\ 2] [1\ 4]$$

$$D_7 = 2^6 q.k_3 q.k_4 (p.k_1)^2 [1\ 2]^2$$

$$D_8 = 2^6 (q.k_4)^2 p.k_1 p.k_2 [1\ 2] [2\ 3]$$

$$D_9 = 2^6 (q.k_4)^2 p.k_1 p.k_3 [1\ 2] [1\ 3]$$

$$D_{10} = 2^6 q.k_1 q.k_4 p.k_1 p.k_2 [1\ 2] [2\ 4]$$

$$D_{11} = 2^6 q.k_2 q.k_4 p.k_1 p.k_4 [1\ 2] [1\ 4]$$

$$D_{12} = 2^6 q.k_1 q.k_4 p.k_1 p.k_3 [1\ 2] [2\ 3]$$

TABLE III (CONTINUED)

$$D_{13} = 2^6 q \cdot k_3 q \cdot k_4 p \cdot k_1 p \cdot k_4 [1\ 2] [2\ 4]$$

$$D_{14} = 2^6 q \cdot k_2 q \cdot k_4 (p \cdot k_1)^2 [1\ 2] [1\ 3]$$

$$D_{15} = 2^6 q \cdot k_3 q \cdot k_4 (p \cdot k_1)^2 [1\ 2] [1\ 4]$$

$$D_{16} = 2^6 q \cdot k_1 q \cdot k_4 p \cdot k_1 p \cdot k_2 [1\ 2] [2\ 3]$$

$$D_{17} = 2^6 q \cdot k_3 q \cdot k_4 p \cdot k_1 p \cdot k_2 [1\ 2] [2\ 4]$$

$$D_{18} = 2^6 q \cdot k_1 q \cdot k_4 p \cdot k_1 p \cdot k_3 [1\ 2] [3\ 4]$$

$$D_{19} = 2^6 q \cdot k_2 q \cdot k_4 p \cdot k_1 p \cdot k_3 [1\ 2] [1\ 3]$$

$$D_{20} = 2^6 q \cdot k_2 q \cdot k_4 p \cdot k_1 p \cdot k_4 [1\ 2] [3\ 4]$$

$$D_{21} = 2^6 q \cdot k_3 q \cdot k_4 p \cdot k_1 p \cdot k_4 [1\ 2] [1\ 4]$$

$$D_{22} = 2^6 q \cdot k_3 q \cdot k_4 p \cdot k_1 p \cdot k_2 [1\ 2]^2$$

$$D_{23} = 2^6 q \cdot k_2 q \cdot k_4 p \cdot k_1 p \cdot k_3 [1\ 2] [3\ 4]$$

$$D_{24} = 2^6 q \cdot k_1 q \cdot k_4 p \cdot k_1 p \cdot k_4 [1\ 2] [3\ 4]$$

$$T_1 = \frac{1}{4m^2} \frac{1}{4} \text{Tr} \left\{ \gamma^\mu [k_4 - \not{d} + m_1] \gamma^\nu [\not{p} - k_1 - k_2 + m_1] \right.$$

$$\gamma^\lambda [\not{p} - k_1 + m_1] \gamma^\rho [\not{p} + m_1] \gamma_\rho [\not{p} - k_1 + m_1] \gamma_\lambda$$

$$\left. [\not{p} - k_1 - k_2 + m_1] \gamma_\nu [k_4 - \not{d} + m_1] \gamma_\mu [\not{d} - m_1] \right\}$$

TABLE III (CONTINUED)

$$T_2 = \frac{1}{4m^2} \frac{1}{4} \text{Tr} \left\{ \gamma^\mu [k_4 - \not{d} + m_1] \gamma^\nu [\not{p} - k_1 - k_2 + m_1] \right. \\ \gamma^\lambda [\not{p} - k_1 + m_1] \gamma^\rho [\not{p} + m_1] \gamma_\lambda [\not{p} - k_2 + m_1] \gamma_\rho \\ \left. [\not{p} - k_1 - k_2 + m_1] \gamma_\nu [k_4 - \not{d} + m_1] \gamma_\mu [\not{d} - m_1] \right\}$$

$$T_3 = \frac{1}{4m^2} \frac{1}{4} \text{Tr} \left\{ \gamma^\mu [k_4 - \not{d} + m_1] \gamma^\nu [\not{p} - k_1 - k_2 + m_1] \right. \\ \gamma^\lambda [\not{p} - k_1 + m_1] \gamma^\rho [\not{p} + m_1] \gamma_\nu [\not{p} - k_3 + m_1] \gamma_\lambda \\ \left. [\not{p} - k_3 - k_2 + m_1] \gamma_\rho [k_4 - \not{d} + m_1] \gamma_\mu [\not{d} - m_1] \right\}$$

$$T_4 = \frac{1}{4m^2} \frac{1}{4} \text{Tr} \left\{ \gamma^\mu [k_4 - \not{d} + m_1] \gamma^\nu [\not{p} - k_1 - k_2 + m_1] \gamma^\lambda \right. \\ [\not{p} - k_1 + m_1] \gamma^\rho [\not{p} + m_1] \gamma_\mu [\not{p} - k_4 + m_1] \gamma_\lambda \\ \left. [\not{p} - k_4 - k_2 + m_1] \gamma_\nu [k_1 - \not{d} + m_1] \gamma_\rho [\not{d} - m_1] \right\}$$

$$T_5 = \frac{1}{4m^2} \frac{1}{4} \text{Tr} \left\{ \gamma^\mu [k_4 - \not{d} + m_1] \gamma^\nu [\not{p} - k_1 - k_2 + m_1] \gamma^\lambda \right. \\ [\not{p} - k_1 + m_1] \gamma^\rho [\not{p} + m_1] \gamma_\rho [\not{p} - k_1 + m_1] \gamma_\nu \\ \left. [\not{p} - k_1 - k_3 + m_1] \gamma_\lambda [k_4 - \not{d} + m_1] \gamma_\mu [\not{d} - m_1] \right\}$$

TABLE III (CONTINUED)

$$T_6 = \frac{1}{4m^2} \frac{1}{4} \text{Tr} \left\{ \gamma^\mu [k_4 - \not{d} + m_1] \gamma^\nu [\not{p} - k_1 - k_2 + m_1] \gamma^\lambda \right. \\ \left. [\not{p} - k_1 + m_1] \gamma^\rho [\not{p} + m_1] \gamma_\rho [\not{p} - k_1 + m_1] \gamma_\mu \right. \\ \left. [\not{p} - k_1 - k_4 + m_1] \gamma_\nu [k_2 - \not{d} + m_1] \gamma_\lambda [\not{d} - m_1] \right\}$$

$$T_7 = \frac{1}{4m^2} \frac{1}{4} \text{Tr} \left\{ \gamma^\mu [k_4 - \not{d} + m_1] \gamma^\nu [\not{p} - k_1 - k_2 + m_1] \gamma^\lambda \right. \\ \left. [\not{p} - k_1 + m_1] \gamma^\rho [\not{p} + m_1] \gamma_\rho [\not{p} - k_1 + m_1] \gamma_\lambda \right. \\ \left. [\not{p} - k_1 - k_2 + m_1] \gamma_\mu [k_3 - \not{d} + m_1] \gamma_\nu [\not{d} - m_1] \right\}$$

$$T_8 = \frac{1}{4m^2} \frac{1}{4} \text{Tr} \left\{ \gamma^\mu [k_4 - \not{d} + m_1] \gamma^\nu [\not{p} - k_1 - k_2 + m_1] \right. \\ \left. \gamma^\lambda [\not{p} - k_1 + m_1] \gamma^\rho [\not{p} + m_1] \gamma_\lambda [\not{p} - k_2 + m_1] \gamma_\nu \right. \\ \left. [\not{p} - k_2 - k_3 + m_1] \gamma_\rho [k_4 - \not{d} + m_1] \gamma_\mu [\not{d} - m_1] \right\}$$

$$T_9 = \frac{1}{4m^2} \frac{1}{4} \text{Tr} \left\{ \gamma^\mu (k_4 - \not{d} + m_1) \gamma^\nu (\not{p} - k_1 - k_2 + m_1) \gamma^\lambda \right. \\ \left. (\not{p} - k_1 + m_1) \gamma^\rho (\not{p} + m_1) \gamma_\nu (\not{p} - k_3 + m_1) \gamma_\rho \right. \\ \left. (\not{p} - k_3 - k_1 + m_1) \gamma_\lambda (k_4 - \not{d} + m_1) \gamma_\mu (\not{d} - m_1) \right\}$$

$$T_{10} = \frac{1}{4m^2} \frac{1}{4} \text{Tr} \left\{ \gamma^\mu [k_4 - \not{d} + m_1] \gamma^\nu [\not{p} - k_1 - k_2 + m_1] \gamma^\lambda \right. \\ \left. [\not{p} - k_1 + m_1] \gamma^\rho [\not{p} + m_1] \gamma_\lambda [\not{p} - k_2 + m_1] \gamma_\mu \right. \\ \left. [\not{p} - k_2 - k_4 + m_1] \gamma_\nu [k_1 - \not{d} + m_1] \gamma_\rho [\not{d} - m_1] \right\}$$

TABLE III (CONTINUED)

$$T_{11} = \frac{1}{4m^2} \frac{1}{4} \text{Tr} \left\{ \gamma^\mu [k_4 - \not{d} + m_1] \gamma^\nu [\not{p} - k_1 - k_2 + m_1] \right. \\ \gamma^\lambda [\not{p} - k_1 + m_1] \gamma^\rho [\not{p} + m_1] \gamma_\mu [\not{p} - k_4 + m_1] \gamma_\rho \\ \left. [\not{p} - k_4 - k_1 + m_1] \gamma_\nu [k_2 - \not{d} + m_1] \gamma_\lambda [\not{d} - m_1] \right\}$$

$$T_{12} = \frac{1}{4m^2} \frac{1}{4} \text{Tr} \left\{ \gamma^\mu [k_4 - \not{d} + m_1] \gamma^\nu [\not{p} - k_1 - k_2 + m_1] \right. \\ \gamma^\lambda [\not{p} - k_1 + m_1] \gamma^\rho [\not{p} + m_1] \gamma_\nu [\not{p} - k_3 + m_1] \gamma_\lambda \\ \left. [\not{p} - k_3 - k_2 + m_1] \gamma_\mu [k_1 - \not{d} + m_1] \gamma_\rho [\not{d} - m_1] \right\}$$

$$T_{13} = \frac{1}{4m^2} \frac{1}{4} \text{Tr} \left\{ \gamma^\mu [k_4 - \not{d} + m_1] \gamma^\nu [\not{p} - k_1 - k_2 + m_1] \right. \\ \gamma^\lambda [\not{p} - k_1 + m_1] \gamma^\rho [\not{p} + m_1] \gamma_\mu [\not{p} - k_4 + m_1] \gamma_\lambda \\ \left. [\not{p} - k_4 - k_2 + m_1] \gamma_\rho [k_3 - \not{d} + m_1] \gamma_\nu [\not{d} - m_1] \right\}$$

$$T_{14} = \frac{1}{4m^2} \frac{1}{4} \text{Tr} \left\{ \gamma^\mu [k_4 - \not{d} + m_1] \gamma^\nu [\not{p} - k_1 - k_2 + m_1] \right. \\ \gamma^\lambda [\not{p} - k_1 + m_1] \gamma^\rho [\not{p} + m_1] \gamma_\rho [\not{p} - k_1 + m_1] \gamma_\nu \\ \left. [\not{p} - k_1 - k_3 + m_1] \gamma_\mu [k_2 - \not{d} + m_1] \gamma_\lambda [\not{d} - m_1] \right\}$$

$$T_{15} = \frac{1}{4m^2} \frac{1}{4} \text{Tr} \left\{ \gamma^\mu [k_4 - \not{d} + m_1] \gamma^\nu [\not{p} - k_1 - k_2 + m_1] \right. \\ \gamma^\lambda [\not{p} - k_1 + m_1] \gamma^\rho [\not{p} + m_1] \gamma_\nu [\not{p} - k_1 + m_1] \gamma_\mu \\ \left. [\not{p} - k_1 - k_4 + m_1] \gamma_\lambda [k_3 - \not{d} + m_1] \gamma_\nu [\not{d} - m_1] \right\}$$

TABLE III (CONTINUED)

$$T_{16} = \frac{1}{4m^2} \frac{1}{4} \text{Tr} \left\{ \gamma^\mu [k_4 - \not{d} + m_1] \gamma^\nu [\not{p} - k_1 - k_2 + m_1] \gamma^\lambda \right. \\ \left. [\not{p} - k_1 + m_1] \gamma^\rho [\not{p} + m_1] \gamma_\lambda [\not{p} - k_2 + m_1] \gamma_\nu \right. \\ \left. [\not{p} - k_2 - k_3 + m_1] \gamma_\mu [k_1 - \not{d} + m_1] \gamma_\rho [\not{d} - m_1] \right\}$$

$$T_{17} = \frac{1}{4m^2} \frac{1}{4} \text{Tr} \left\{ \gamma^\mu [k_4 - \not{d} + m_1] \gamma^\nu [\not{p} - k_1 - k_2 + m_1] \gamma^\lambda \right. \\ \left. [\not{p} - k_1 + m_1] \gamma^\rho [\not{p} + m_1] \gamma_\lambda [\not{p} - k_2 + m_1] \gamma_\mu \right. \\ \left. [\not{p} - k_2 - k_4 + m_1] \gamma_\rho [k_3 - \not{d} + m_1] \gamma_\nu [\not{d} - m_1] \right\}$$

$$T_{18} = \frac{1}{4m^2} \frac{1}{4} \text{Tr} \left\{ \gamma^\mu [k_4 - \not{d} + m_1] \gamma^\nu [\not{p} - k_1 - k_2 + m_1] \gamma^\lambda \right. \\ \left. [\not{p} - k_1 + m_1] \gamma^\rho [\not{p} + m_1] \gamma_\nu [\not{p} - k_3 + m_1] \gamma_\mu \right. \\ \left. [\not{p} - k_3 - k_4 + m_1] \gamma_\lambda [k_1 - \not{d} + m_1] \gamma_\rho [\not{d} - m_1] \right\}$$

$$T_{19} = \frac{1}{4m^2} \frac{1}{4} \text{Tr} \left\{ \gamma^\mu [k_4 - \not{d} + m_1] \gamma^\nu [\not{p} - k_1 - k_2 + m_1] \right. \\ \left. \gamma^\lambda [\not{p} - k_1 + m_1] \gamma^\rho [\not{p} + m_1] \gamma_\nu [\not{p} - k_3 + m_1] \gamma_\rho \right. \\ \left. [\not{p} - k_3 - k_1 + m_1] \gamma_\mu [k_2 - \not{d} + m_1] \gamma_\lambda [\not{d} - m_1] \right\}$$

$$T_{20} = \frac{1}{4m^2} \frac{1}{4} \text{Tr} \left\{ \gamma^\mu [k_4 - \not{d} + m_1] \gamma^\nu [\not{p} - k_1 - k_2 + m_1] \right. \\ \left. \gamma^\lambda [\not{p} - k_1 + m_1] \gamma^\rho [\not{p} + m_1] \gamma_\mu [\not{p} - k_4 + m_1] \gamma_\nu \right. \\ \left. [\not{p} - k_4 - k_3 + m_1] \gamma_\rho [k_2 - \not{d} + m_1] \gamma_\lambda [\not{d} - m_1] \right\}$$

TABLE III (CONTINUED)

$$T_{21} = \frac{1}{4m^2} \frac{1}{4} \text{Tr} \left\{ \gamma^\mu [k_4 - \not{d} + m_1] \gamma^\nu [\not{p} - k_1 - k_2 + m_1] \right. \\ \gamma^\lambda [\not{p} - k_1 + m_1] \gamma^\rho [\not{p} + m_1] \gamma_\mu [\not{p} - k_4 + m_1] \gamma_\rho \\ \left. [\not{p} - k_4 - k_1 + m_1] \gamma_\lambda [k_3 - \not{d} + m_1] \gamma_\nu [\not{d} - m_1] \right\}$$

$$T_{22} = \frac{1}{4m^2} \frac{1}{4} \text{Tr} \left\{ \gamma^\mu [k_4 - \not{d} + m_1] \gamma^\nu [\not{p} - k_1 - k_2 + m_1] \gamma^\lambda \right. \\ [\not{p} - k_1 + m_1] \gamma^\rho [\not{p} + m_1] \gamma_\lambda [\not{p} - k_2 + m_1] \gamma_\rho \\ \left. [\not{p} - k_1 - k_2 + m_1] \gamma_\mu [k_3 - \not{d} + m_1] \gamma_\nu [\not{d} - m_1] \right\}$$

$$T_{23} = \frac{1}{4m^2} \frac{1}{4} \text{Tr} \left\{ \gamma^\mu [k_4 - \not{d} + m_1] \gamma^\nu [\not{p} - k_1 - k_2 + m_1] \right. \\ \gamma^\lambda [\not{p} - k_1 + m_1] \gamma^\rho [\not{p} + m_1] \gamma_\nu [\not{p} - k_3 + m_1] \gamma_\mu \\ \left. [\not{p} - k_3 - k_4 + m_1] \gamma_\rho [k_2 - \not{d} + m_1] \gamma_\lambda [\not{d} - m_1] \right\}$$

$$T_{24} = \frac{1}{4m^2} \frac{1}{4} \text{Tr} \left\{ \gamma^\mu [k_4 - \not{d} + m_1] \gamma^\nu [\not{p} - k_1 - k_2 + m_1] \gamma^\lambda \right. \\ [\not{p} - k_1 + m_1] \gamma^\rho [\not{p} + m_1] \gamma_\mu [\not{p} - k_4 + m_1] \gamma_\nu \\ \left. [\not{p} - k_3 - k_4 + m_1] \gamma_\lambda [k_1 - \not{d} + m_1] \gamma_\rho [\not{d} - m_1] \right\}$$

TABLE IV

REDUCED CROSS SECTION* $e^+ + e^- \rightarrow 4\gamma$

$$\begin{aligned}
\sigma_r = 24 A \frac{1}{4m^2} \frac{1}{2^6} \{ & \\
+768 P_2^{-1} Q_4^{-1} & \\
+128 p.k_4 P_1^{-1} P_2^{-1} Q_4^{-1} & \\
+128 p.q P_1^{-1} P_2^{-1} Q_4^{-1} & \\
+128 p.k_3 P_1^{-1} P_2^{-1} Q_4^{-1} & \\
-128 p.q P_1^{-1} Q_3^{-1} Q_4^{-1} & \\
-128 p.q P_1^{-1} Q_1^{-1} Q_4^{-1} & \\
+384 q.k_1 P_1^{-1} Q_2^{-1} Q_4^{-1} & \\
+384 q.k_2 P_1^{-1} Q_3^{-1} Q_4^{-1} & \\
+128 (p.q)^2 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} & \\
-128 p.q p.k_4 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} & \\
+256 p.k_3 q.k_1 P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} & \\
-128 p.q p.k_3 P_1^{-1} P_2^{-1} Q_1^{-1} Q_4^{-1} & \\
-192 (p.q)^2 P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} & \\
\} &
\end{aligned}$$

* with $P_i \equiv p.k_i$, $Q_i \equiv q.k_i$

TABLE IV (CONTINUED)

$$-128 p.q [1\ 2] P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1}$$

$$-128 p.q k_1.k_2 P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1}$$

$$-192 P_1^{-1} [1\ 2]^{-1}$$

$$-256 P_1^{-1} [2\ 3]^{-1}$$

$$-32 Q_4^{-1} [1\ 2]^{-1}$$

$$-128 Q_1^{-1} [1\ 2]^{-1}$$

$$-128 p.k_4 P_1^{-1} P_3^{-1} [1\ 2]^{-1}$$

$$-640 p.q P_1^{-1} P_4^{-1} [1\ 2]^{-1}$$

$$-384 p.k_4 P_1^{-1} Q_4^{-1} [1\ 2]^{-1}$$

$$-320 p.k_4 P_3^{-1} Q_4^{-1} [1\ 2]^{-1}$$

$$-128 p.q P_1^{-1} Q_4^{-1} [2\ 4]^{-1}$$

$$+128 p.k_2 P_1^{-1} Q_2^{-1} [1\ 2]^{-1}$$

$$+128 p.q P_1^{-1} Q_1^{-1} [1\ 2]^{-1}$$

$$+256 p.q P_1^{-1} Q_1^{-1} [2\ 3]^{-1}$$

$$+128 p.k_4 P_1^{-1} Q_4^{-1} [2\ 4]^{-1}$$

TABLE IV (CONTINUED)

$$+128 p.k_2 P_1^{-1} Q_4^{-1} [1\ 2]^{-1}$$

$$-128 p.k_3 P_2^{-1} Q_4^{-1} [1\ 2]^{-1}$$

$$+128 p.k_2 P_3^{-1} Q_1^{-1} [1\ 2]^{-1}$$

$$+256 p.q P_1^{-1} Q_2^{-1} [1\ 2]^{-1}$$

$$+384 p.q P_1^{-1} Q_4^{-1} [2\ 3]^{-1}$$

$$+128 p.k_1 P_2^{-1} Q_4^{-1} [2\ 4]^{-1}$$

$$-256 q.k_3 P_2^{-1} Q_4^{-1} [1\ 2]^{-1}$$

$$+128 q.k_2 P_1^{-1} Q_4^{-1} [1\ 2]^{-1}$$

$$+128 q.k_1 P_1^{-1} Q_4^{-1} [1\ 2]^{-1}$$

$$+64 q.k_3 P_1^{-1} Q_4^{-1} [2\ 3]^{-1}$$

$$-64 q.k_3 P_3^{-1} Q_1^{-1} [1\ 2]^{-1}$$

$$+64 q.k_1 P_3^{-1} Q_2^{-1} [1\ 2]^{-1}$$

$$-128 k_2.k_4 P_1^{-1} Q_4^{-1} [1\ 2]^{-1}$$

$$+128 [1\ 2] P_1^{-1} Q_4^{-1} [2\ 3]^{-1}$$

$$-64 k_1.k_3 P_3^{-1} Q_4^{-1} [1\ 2]^{-1}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& +128 \, p \cdot q \, Q_1^{-1} \, Q_4^{-1} \, [1 \, 2]^{-1} \\
& -256 \, q \cdot k_1 \, Q_2^{-1} \, Q_4^{-1} \, [1 \, 2]^{-1} \\
& -128 \, q \cdot k_3 \, Q_2^{-1} \, Q_4^{-1} \, [1 \, 2]^{-1} \\
& +896 \, (p \cdot q)^2 \, P_1^{-1} \, P_4^{-1} \, Q_1^{-1} \, [1 \, 2]^{-1} \\
& +128 \, p \cdot q \, p \cdot k_3 \, P_1^{-1} \, P_2^{-1} \, Q_4^{-1} \, [2 \, 3]^{-1} \\
& -128 \, p \cdot q \, p \cdot k_2 \, P_1^{-1} \, P_3^{-1} \, Q_1^{-1} \, [1 \, 2]^{-1} \\
& -128 \, p \cdot k_3 \, p \cdot k_4 \, P_1^{-1} \, P_2^{-1} \, Q_4^{-1} \, [2 \, 3]^{-1} \\
& -128 \, p \cdot q \, p \cdot k_4 \, P_1^{-1} \, P_3^{-1} \, Q_1^{-1} \, [1 \, 2]^{-1} \\
& +512 \, (p \cdot q)^2 \, P_1^{-1} \, P_4^{-1} \, Q_4^{-1} \, [1 \, 2]^{-1} \\
& +256 \, p \cdot q \, p \cdot k_2 \, P_1^{-1} \, P_4^{-1} \, Q_1^{-1} \, [2 \, 4]^{-1} \\
& +192 \, (p \cdot q)^2 \, P_1^{-1} \, P_2^{-1} \, Q_4^{-1} \, [2 \, 4]^{-1} \\
& -128 \, p \cdot q \, p \cdot k_4 \, P_1^{-1} \, P_3^{-1} \, Q_1^{-1} \, [2 \, 3]^{-1} \\
& +128 \, (p \cdot q)^2 \, P_1^{-1} \, P_2^{-1} \, Q_1^{-1} \, [2 \, 3]^{-1} \\
& +448 \, (p \cdot q)^2 \, P_1^{-1} \, P_3^{-1} \, Q_4^{-1} \, [1 \, 2]^{-1} \\
& -384 \, p \cdot q \, q \cdot k_4 \, P_1^{-1} \, P_4^{-1} \, Q_1^{-1} \, [2 \, 4]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$-256 p.q q.k_2 P_1^{-1} P_4^{-1} Q_1^{-1} [1\ 2]^{-1}$$

$$-64 p.q q.k_2 P_1^{-1} P_2^{-1} Q_4^{-1} [2\ 4]^{-1}$$

$$-320 p.q q.k_2 P_1^{-1} P_2^{-1} Q_4^{-1} [2\ 3]^{-1}$$

$$+128 p.k_3 q.k_2 P_1^{-1} P_2^{-1} Q_3^{-1} [2\ 4]^{-1}$$

$$-128 p.q q.k_3 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1}$$

$$-384 p.q q.k_2 P_1^{-1} P_4^{-1} Q_4^{-1} [1\ 2]^{-1}$$

$$+128 p.k_3 q.k_2 P_1^{-1} P_2^{-1} Q_4^{-1} [2\ 3]^{-1}$$

$$-128 p.q q.k_3 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1}$$

$$+128 p.k_4 q.k_2 P_1^{-1} P_2^{-1} Q_4^{-1} [2\ 4]^{-1}$$

$$-128 p.q q.k_2 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1}$$

$$-128 p.q k_2.k_3 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1}$$

$$-32 p.q k_1.k_3 P_1^{-1} P_3^{-1} Q_2^{-1} [1\ 2]^{-1}$$

$$+128 p.k_3 [1\ 2] P_1^{-1} P_2^{-1} Q_4^{-1} [2\ 3]^{-1}$$

$$-288 p.q k_1.k_3 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1}$$

$$+128 p.q p.k_2 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& +448 (p.q)^2 P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +128 p.q p.k_3 P_1^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +256 p.q p.k_4 P_2^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& -448 (p.q)^2 P_2^{-1} Q_3^{-1} Q_4^{-1} [2\ 4]^{-1} \\
& -128 (p.q)^2 P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +256 p.q p.k_1 P_3^{-1} Q_1^{-1} Q_4^{-1} [2\ 3]^{-1} \\
& -128 p.q p.k_1 P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +384 (p.q)^2 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +448 (p.q)^2 P_3^{-1} Q_1^{-1} Q_4^{-1} [2\ 3]^{-1} \\
& +192 p.q p.k_1 P_2^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& -128 p.q p.k_2 P_1^{-1} Q_3^{-1} Q_4^{-1} [2\ 4]^{-1} \\
& -64 p.q p.k_4 P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +192 p.q q.k_1 P_1^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +128 p.q q.k_2 P_1^{-1} Q_1^{-1} Q_4^{-1} [2\ 4]^{-1} \\
& -256 p.k_2 q.k_1 P_1^{-1} Q_2^{-1} Q_4^{-1} [1\ 3]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& +256 p \cdot q \cdot q \cdot k_3 P_2^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& +128 p \cdot q \cdot q \cdot k_3 P_1^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& +256 p \cdot k_2 q \cdot k_3 P_3^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& -320 p \cdot q \cdot q \cdot k_3 P_3^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& -128 p \cdot q \cdot q \cdot k_2 P_1^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& -576 p \cdot q \cdot q \cdot k_2 P_3^{-1} Q_1^{-1} Q_4^{-1} [2 \ 3]^{-1} \\
& -128 p \cdot k_4 q \cdot k_1 P_1^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& -128 p \cdot k_3 q \cdot k_1 P_2^{-1} Q_3^{-1} Q_4^{-1} [2 \ 4]^{-1} \\
& -256 p \cdot k_3 q \cdot k_2 P_1^{-1} Q_3^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& -576 p \cdot q \cdot q \cdot k_1 P_1^{-1} Q_3^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& -64 p \cdot k_2 q \cdot k_1 P_1^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& +64 p \cdot k_4 q \cdot k_3 P_3^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& +128 (q \cdot k_1)^2 P_1^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& -128 p \cdot q [2 \ 3] P_3^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& +32 p \cdot q k_1 \cdot k_3 P_1^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& +224 \, p \cdot q \, k_1 \cdot k_3 \, P_3^{-1} \, Q_2^{-1} \, Q_4^{-1} \, [1 \, 2]^{-1} \\
& -128 \, q \cdot k_1 \, [1 \, 3] \, P_1^{-1} \, Q_2^{-1} \, Q_4^{-1} \, [1 \, 2]^{-1} \\
& -128 \, p \cdot q \, (p \cdot k_2)^2 \, P_1^{-1} \, P_4^{-1} \, Q_1^{-1} \, Q_4^{-1} \, [1 \, 2]^{-1} \\
& -384 \, (p \cdot q)^2 \, p \cdot k_2 \, P_1^{-1} \, P_4^{-1} \, Q_1^{-1} \, Q_4^{-1} \, [1 \, 2]^{-1} \\
& -384 \, (p \cdot q)^3 \, P_1^{-1} \, P_3^{-1} \, Q_1^{-1} \, Q_4^{-1} \, [1 \, 2]^{-1} \\
& -256 \, (p \cdot q)^3 \, P_1^{-1} \, P_3^{-1} \, Q_2^{-1} \, Q_4^{-1} \, [1 \, 2]^{-1} \\
& -512 \, (p \cdot q)^3 \, P_1^{-1} \, P_4^{-1} \, Q_1^{-1} \, Q_4^{-1} \, [1 \, 2]^{-1} \\
& +128 \, p \cdot q \, p \cdot k_2 \, p \cdot k_4 \, P_1^{-1} \, P_3^{-1} \, Q_1^{-1} \, Q_4^{-1} \, [1 \, 2]^{-1} \\
& +128 \, (p \cdot q)^2 \, p \cdot k_4 \, P_1^{-1} \, P_3^{-1} \, Q_1^{-1} \, Q_4^{-1} \, [1 \, 2]^{-1} \\
& +128 \, (p \cdot q)^2 \, p \cdot k_4 \, P_1^{-1} \, P_3^{-1} \, Q_2^{-1} \, Q_4^{-1} \, [1 \, 2]^{-1} \\
& +384 \, (p \cdot q)^2 \, q \cdot k_2 \, P_1^{-1} \, P_4^{-1} \, Q_1^{-1} \, Q_4^{-1} \, [1 \, 2]^{-1} \\
& +128 \, (p \cdot q)^2 \, q \cdot k_2 \, P_1^{-1} \, P_2^{-1} \, Q_1^{-1} \, Q_4^{-1} \, [2 \, 4]^{-1} \\
& -128 \, p \cdot q \, p \cdot k_2 \, q \cdot k_2 \, P_1^{-1} \, P_3^{-1} \, Q_1^{-1} \, Q_4^{-1} \, [1 \, 2]^{-1} \\
& -192 \, p \cdot q \, p \cdot k_2 \, q \cdot k_1 \, P_1^{-1} \, P_3^{-1} \, Q_2^{-1} \, Q_4^{-1} \, [1 \, 2]^{-1} \\
& +256 \, p \cdot q \, p \cdot k_2 \, q \cdot k_2 \, P_1^{-1} \, P_4^{-1} \, Q_1^{-1} \, Q_4^{-1} \, [1 \, 2]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& +192 p \cdot q p \cdot k_4 q \cdot k_1 P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [2 \ 4]^{-1} \\
& +128 (p \cdot q)^2 q \cdot k_3 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& +384 (p \cdot q)^2 q \cdot k_1 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& -128 p \cdot q (q \cdot k_2)^2 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& -256 p \cdot k_3 (q \cdot k_1)^2 P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& +128 p \cdot q (q \cdot k_1)^2 P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& +64 p \cdot k_2 (q \cdot k_1)^2 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& -64 p \cdot k_2 (q \cdot k_3)^2 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& -128 p \cdot q (q \cdot k_2)^2 P_1^{-1} P_2^{-1} Q_1^{-1} Q_4^{-1} [2 \ 4]^{-1} \\
& -256 p \cdot k_3 q \cdot k_1 q \cdot k_2 P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& +128 p \cdot q q \cdot k_1 q \cdot k_2 P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& -64 p \cdot k_2 q \cdot k_1 q \cdot k_3 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& +64 p \cdot k_4 q \cdot k_1 q \cdot k_3 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& -128 p \cdot k_3 q \cdot k_2 [1 \ 2] P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [2 \ 4]^{-1} \\
& +128 p \cdot q q \cdot k_2 [1 \ 2] P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [2 \ 4]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$+64 p \cdot k_2 q \cdot k_1 k_1 \cdot k_3 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1}$$

$$-256 p \cdot q q \cdot k_1 k_1 \cdot k_3 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1}$$

$$-128 p \cdot q p \cdot k_4 [1 \ 2] P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [2 \ 4]^{-1}$$

$$-64 p \cdot q (k_1 \cdot k_3)^2 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1}$$

$$-64 p \cdot q p \cdot k_2 k_1 \cdot k_3 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1}$$

$$+256 (p \cdot q)^2 k_1 \cdot k_3 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1}$$

$$+128 [1 \ 2]^{-1} [2 \ 3]^{-1}$$

$$+96 [1 \ 2]^{-1} [3 \ 4]^{-1}$$

$$+128 p \cdot k_4 P_3^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1}$$

$$+128 p \cdot k_4 P_1^{-1} [1 \ 2]^{-1} [1 \ 3]^{-1}$$

$$+128 p \cdot k_2 P_1^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1}$$

$$+256 p \cdot q P_2^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1}$$

$$+128 p \cdot q P_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1}$$

$$-128 p \cdot q P_4^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1}$$

$$-256 p \cdot k_3 P_1^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1}$$

TABLE IV (CONTINUED)

$$+128 p.k_3 P_2^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$+192 p.k_4 P_3^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-128 q.k_1 P_1^{-1} [1\ 2]^{-1} [1\ 3]^{-1}$$

$$+128 q.k_4 P_1^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-64 q.k_1 P_1^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+128 q.k_1 P_1^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$+256 p.k_4 Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-320 p.q Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$+64 p.k_4 Q_2^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+128 p.k_4 Q_2^{-1} [1\ 2]^{-1} [1\ 3]^{-1}$$

$$-32 p.k_2 Q_1^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-384 p.q Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+128 q.k_3 Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-128 q.k_1 Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$-192 q.k_1 Q_4^{-1} [1\ 2]^{-1} [1\ 4]^{-1}$$

TABLE IV (CONTINUED)

$$+128 p \cdot k_2 p \cdot k_4 P_1^{-1} P_3^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1}$$

$$+128 (p \cdot k_2)^2 P_1^{-1} P_3^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1}$$

$$+128 (p \cdot q)^2 P_1^{-1} P_3^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1}$$

$$-128 (p \cdot q)^2 P_1^{-1} P_3^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1}$$

$$+128 p \cdot k_4 q \cdot k_4 P_1^{-1} P_3^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1}$$

$$+128 p \cdot k_2 q \cdot k_4 P_1^{-1} P_3^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1}$$

$$-128 p \cdot q q \cdot k_3 P_1^{-1} P_4^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1}$$

$$-256 p \cdot q q \cdot k_3 P_1^{-1} P_3^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1}$$

$$-64 p \cdot q k_1 \cdot k_4 P_1^{-1} P_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1}$$

$$+512 p \cdot k_2 p \cdot k_4 P_3^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1}$$

$$+192 p \cdot q p \cdot k_2 P_4^{-1} Q_1^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1}$$

$$+128 p \cdot k_3 p \cdot k_4 P_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1}$$

$$+128 (p \cdot k_2)^2 P_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1}$$

$$-128 (p \cdot q)^2 P_2^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1}$$

$$+384 p \cdot q p \cdot k_4 P_3^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1}$$

TABLE IV (CONTINUED)

$$-128 p \cdot q p \cdot k_3 P_1^{-1} Q_1^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1}$$

$$-128 p \cdot q p \cdot k_1 P_2^{-1} Q_1^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1}$$

$$-384 p \cdot q p \cdot k_2 P_1^{-1} Q_1^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1}$$

$$+192 p \cdot q p \cdot k_4 P_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1}$$

$$-192 (p \cdot q)^2 P_4^{-1} Q_1^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1}$$

$$-128 (p \cdot q)^2 P_4^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1}$$

$$+128 p \cdot q p \cdot k_3 P_2^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1}$$

$$+128 p \cdot q p \cdot k_3 P_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1}$$

$$+192 p \cdot q p \cdot k_4 P_3^{-1} Q_1^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1}$$

$$+128 p \cdot q p \cdot k_2 P_3^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1}$$

$$-128 p \cdot q p \cdot k_3 P_1^{-1} Q_2^{-1} [1 \ 2]^{-1} [1 \ 3]^{-1}$$

$$-256 (p \cdot q)^2 P_2^{-1} Q_1^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1}$$

$$-128 (p \cdot q)^2 P_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1}$$

$$-128 (p \cdot q)^2 P_2^{-1} Q_3^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1}$$

$$-128 p \cdot q p \cdot k_3 P_2^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1}$$

TABLE IV (CONTINUED)

$$-128 p \cdot q p \cdot k_2 P_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1}$$

$$-128 (p \cdot q)^2 P_3^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1}$$

$$+576 (p \cdot q)^2 P_4^{-1} Q_1^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1}$$

$$+64 (p \cdot k_4)^2 P_3^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1}$$

$$-64 p \cdot q p \cdot k_4 P_3^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1}$$

$$-256 p \cdot q p \cdot k_4 P_1^{-1} Q_1^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1}$$

$$+192 p \cdot q p \cdot k_4 P_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1}$$

$$+192 (p \cdot q)^2 P_1^{-1} Q_1^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1}$$

$$-128 p \cdot k_1 p \cdot k_4 P_3^{-1} Q_2^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1}$$

$$-192 p \cdot q p \cdot k_1 P_3^{-1} Q_2^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1}$$

$$-128 p \cdot k_2 q \cdot k_3 P_1^{-1} Q_4^{-1} [1 \ 2]^{-2}$$

$$+128 p \cdot q q \cdot k_2 P_4^{-1} Q_1^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1}$$

$$+128 p \cdot k_2 q \cdot k_3 P_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1}$$

$$-128 p \cdot k_3 q \cdot k_2 P_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1}$$

$$-256 p \cdot k_2 q \cdot k_2 P_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& -128 p.k_2 q.k_1 P_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& -128 p.k_4 q.k_3 P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& +256 p.q q.k_3 P_2^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -64 p.k_3 q.k_2 P_2^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& +128 p.k_4 q.k_3 P_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -128 p.q q.k_3 P_1^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -128 p.q q.k_4 P_4^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +960 p.q q.k_1 P_1^{-1} Q_4^{-1} [1\ 2]^{-1} [1\ 4]^{-1} \\
& +128 p.k_3 q.k_2 P_2^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -128 p.k_3 q.k_1 P_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -128 p.k_4 q.k_2 P_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& -256 p.k_4 q.k_1 P_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& -128 p.q q.k_3 P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -320 p.k_1 q.k_2 P_2^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -128 p.k_4 q.k_1 P_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& -256 p \cdot q \, q \cdot k_3 \, P_1^{-1} \, Q_4^{-1} \, [1 \, 2]^{-1} \, [2 \, 3]^{-1} \\
& -256 p \cdot q \, q \cdot k_4 \, P_1^{-1} \, Q_1^{-1} \, [1 \, 2]^{-1} \, [2 \, 3]^{-1} \\
& -128 p \cdot k_4 \, q \cdot k_1 \, P_2^{-1} \, Q_4^{-1} \, [1 \, 2]^{-1} \, [2 \, 4]^{-1} \\
& +128 p \cdot k_2 \, q \cdot k_3 \, P_1^{-1} \, Q_4^{-1} \, [1 \, 2]^{-1} \, [2 \, 4]^{-1} \\
& -320 p \cdot k_3 \, q \cdot k_2 \, P_2^{-1} \, Q_4^{-1} \, [1 \, 2]^{-1} \, [2 \, 4]^{-1} \\
& -128 p \cdot q \, q \cdot k_3 \, P_3^{-1} \, Q_1^{-1} \, [1 \, 2]^{-1} \, [3 \, 4]^{-1} \\
& -64 p \cdot q \, q \cdot k_2 \, P_1^{-1} \, Q_1^{-1} \, [1 \, 2]^{-1} \, [3 \, 4]^{-1} \\
& +64 p \cdot k_4 \, q \cdot k_3 \, P_3^{-1} \, Q_2^{-1} \, [1 \, 2]^{-1} \, [3 \, 4]^{-1} \\
& -384 p \cdot k_2 \, q \cdot k_3 \, P_3^{-1} \, Q_4^{-1} \, [1 \, 2]^{-1} \, [3 \, 4]^{-1} \\
& +64 p \cdot k_4 \, q \cdot k_2 \, P_3^{-1} \, Q_4^{-1} \, [1 \, 2]^{-1} \, [3 \, 4]^{-1} \\
& +128 p \cdot k_2 \, q \cdot k_1 \, P_1^{-1} \, Q_3^{-1} \, [1 \, 2]^{-1} \, [2 \, 4]^{-1} \\
& +256 p \cdot q \, q \cdot k_1 \, P_2^{-1} \, Q_4^{-1} \, [1 \, 2]^{-1} \, [2 \, 4]^{-1} \\
& +448 p \cdot q \, q \cdot k_3 \, P_1^{-1} \, Q_4^{-1} \, [1 \, 2]^{-1} \, [3 \, 4]^{-1} \\
& -64 p \cdot k_4 \, q \cdot k_1 \, P_3^{-1} \, Q_2^{-1} \, [1 \, 2]^{-1} \, [3 \, 4]^{-1} \\
& -256 p \cdot k_2 \, q \cdot k_2 \, P_3^{-1} \, Q_4^{-1} \, [1 \, 2]^{-1} \, [3 \, 4]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$+768 p \cdot q q \cdot k_3 P_3^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1}$$

$$-256 (q \cdot k_1)^2 P_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [1 \ 4]^{-1}$$

$$+128 (q \cdot k_1)^2 P_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1}$$

$$-64 q \cdot k_1 q \cdot k_3 P_3^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1}$$

$$+128 q \cdot k_1 q \cdot k_2 P_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1}$$

$$-128 q \cdot k_1 q \cdot k_2 P_2^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1}$$

$$-128 q \cdot k_2 q \cdot k_3 P_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1}$$

$$+128 p \cdot q k_2 \cdot k_3 P_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1}$$

$$-64 k_1 \cdot k_3 k_2 \cdot k_3 P_3^{-1} Q_2^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1}$$

$$+64 p \cdot k_4 k_1 \cdot k_3 P_3^{-1} Q_2^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1}$$

$$-128 p \cdot q k_2 \cdot k_4 P_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1}$$

$$+128 p \cdot q k_1 \cdot k_3 P_3^{-1} Q_2^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1}$$

$$-128 p \cdot q p \cdot k_3 Q_3^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1}$$

$$+64 (p \cdot q)^2 Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1}$$

$$+64 p \cdot q p \cdot k_1 Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& +128 p.k_4 q.k_1 Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [1\ 3]^{-1} \\
& -128 p.k_4 q.k_3 Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +128 p.k_3 q.k_1 Q_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +256 p.q q.k_3 Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -128 q.k_1 q.k_3 Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [1\ 3]^{-1} \\
& -64 p.q k_1.k_4 Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -32 p.q k_1.k_3 Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +128 (p.k_2)^2 p.k_4 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -256 (p.q)^2 p.k_2 P_1^{-1} P_4^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +128 p.q (p.k_2)^2 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& +128 (p.q)^3 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -64 (p.q)^2 p.k_2 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -256 (p.q)^2 p.k_2 P_1^{-1} P_4^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -256 p.q (p.k_2)^2 P_1^{-1} P_4^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +128 p.k_2 (p.k_4)^2 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& -128 (p.q)^2 p.k_2 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& +128 p.q p.k_2 p.k_4 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -256 (p.q)^3 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -256 (p.q)^2 q.k_3 P_1^{-1} P_4^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +128 p.q p.k_2 q.k_2 P_1^{-1} P_4^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& -128 p.q p.k_4 q.k_3 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& +256 p.q p.k_2 q.k_3 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -256 (p.q)^2 q.k_3 P_1^{-1} P_4^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +128 (p.q)^2 q.k_1 P_1^{-1} P_3^{-1} Q_2^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -256 p.q p.k_2 q.k_4 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -128 p.q p.k_4 q.k_2 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& +128 (p.k_4)^2 q.k_3 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +256 (p.q)^2 q.k_3 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +64 p.q p.k_2 q.k_1 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +128 p.q q.k_2 q.k_3 P_1^{-1} P_4^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 4]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& -128 p \cdot q (q \cdot k_4)^2 P_1^{-1} P_3^{-1} Q_1^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1} \\
& +64 p \cdot k_2 (q \cdot k_3)^2 P_1^{-1} P_3^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& -64 p \cdot q q \cdot k_1 q \cdot k_3 P_1^{-1} P_3^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& +128 p \cdot k_4 q \cdot k_3 k_2 \cdot k_3 P_1^{-1} P_3^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& -128 p \cdot q p \cdot k_4 k_2 \cdot k_3 P_1^{-1} P_3^{-1} Q_4^{-1} [1 \ 2]^{-1} [4 \ 3]^{-1} \\
& -256 p \cdot q q \cdot k_3 k_2 \cdot k_3 P_1^{-1} P_3^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& +256 (p \cdot q)^2 k_2 \cdot k_3 P_1^{-1} P_3^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& +64 p \cdot k_2 (k_1 \cdot k_3)^2 P_1^{-1} P_3^{-1} Q_2^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& +256 p \cdot k_2 q \cdot k_1 k_1 \cdot k_3 P_1^{-1} P_3^{-1} Q_2^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& -224 p \cdot q p \cdot k_2 k_1 \cdot k_3 P_1^{-1} P_3^{-1} Q_2^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& -544 p \cdot q q \cdot k_1 k_1 \cdot k_3 P_1^{-1} P_3^{-1} Q_2^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& +256 (p \cdot q)^2 k_1 \cdot k_3 P_1^{-1} P_3^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& -128 (p \cdot q)^2 p \cdot k_2 P_3^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1} \\
& -128 (p \cdot q)^2 p \cdot k_4 P_1^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1} \\
& +128 (p \cdot q)^2 p \cdot k_1 P_2^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& +128 (p.q)^2 p.k_3 P_2^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& -192 p.q (p.k_4)^2 P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +128 (p.q)^2 p.k_2 P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -128 (p.q)^3 P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -128 (p.q)^2 p.k_2 P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -128 (p.q)^2 p.k_2 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -128 (p.q)^2 p.k_3 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -128 (p.q)^3 P_2^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +256 (p.q)^2 p.k_4 P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -384 (p.q)^3 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -128 p.q p.k_2 p.k_4 P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -64 (p.q)^2 p.k_2 P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +128 p.q p.k_3 q.k_3 P_2^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& +128 p.q p.k_2 q.k_1 P_1^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +128 p.q p.k_3 q.k_1 P_1^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& +128 (p \cdot q)^2 q \cdot k_2 P_2^{-1} Q_3^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1} \\
& +256 p \cdot q p \cdot k_2 q \cdot k_3 P_3^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& -192 (p \cdot q)^2 q \cdot k_3 P_3^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& +256 (p \cdot q)^2 q \cdot k_2 P_4^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& +384 p \cdot q p \cdot k_2 q \cdot k_3 P_3^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& +128 p \cdot q p \cdot k_2 q \cdot k_2 P_3^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1} \\
& +128 (p \cdot q)^2 q \cdot k_2 P_1^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1} \\
& +128 (p \cdot q)^2 q \cdot k_3 P_1^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1} \\
& -128 p \cdot q p \cdot k_3 q \cdot k_2 P_1^{-1} Q_3^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1} \\
& -192 p \cdot k_2 p \cdot k_4 q \cdot k_3 P_3^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& -128 p \cdot q p \cdot k_4 q \cdot k_2 P_1^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& +384 (p \cdot q)^2 q \cdot k_2 P_1^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& -64 p \cdot k_2 p \cdot k_4 q \cdot k_3 P_3^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& +256 p \cdot q p \cdot k_4 q \cdot k_3 P_3^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& -128 p \cdot k_4 q \cdot k_1 q \cdot k_3 P_1^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1} [1 \ 3]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& -128 p.k_4 q.k_1 q.k_2 P_2^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +128 p.k_3 (q.k_2)^2 P_1^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +256 p.k_2 (q.k_1)^2 P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +576 p.q q.k_1 q.k_3 P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +128 p.k_3 (q.k_1)^2 P_1^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [1\ 3]^{-1} \\
& -128 p.q (q.k_1)^2 P_1^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [1\ 3]^{-1} \\
& -128 p.k_3 (q.k_1)^2 P_1^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +128 p.k_4 (q.k_3)^2 P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -64 p.q (q.k_1)^2 P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +128 p.k_4 q.k_3 k_2.k_3 P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -160 p.q p.k_4 k_2.k_3 P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +32 p.q q.k_3 k_2.k_3 P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +64 q.k_3 k_1.k_3 k_2.k_4 P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -64 p.k_4 q.k_1 k_1.k_3 P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +256 (p.q)^4 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& +128 (p.q)^4 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -64 (p.q)^2 (p.k_2)^2 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -64 (p.q)^4 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +64 p.q (p.k_2)^3 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +128 (p.q)^3 p.k_2 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +128 (p.q)^2 (p.k_2)^2 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +128 (p.q)^2 p.k_4 q.k_3 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -512 (p.q)^3 q.k_3 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +64 p.q p.k_2 p.k_4 q.k_1 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +256 (p.q)^2 p.k_2 q.k_1 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +64 p.q (p.k_2)^2 q.k_3 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +128 (p.q)^3 q.k_3 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +128 (p.q)^2 p.k_2 q.k_3 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& -128 (p.q)^2 p.k_2 q.k_2 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& -128 p.q (p.k_2)^2 q.k_2 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& -128 p \cdot q p \cdot k_4 (q \cdot k_3)^2 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& +256 (p \cdot q)^2 (q \cdot k_3)^2 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& +64 p \cdot k_2 p \cdot k_4 (q \cdot k_1)^2 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& -64 (p \cdot q)^2 (q \cdot k_1)^2 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& +96 (p \cdot q)^2 q \cdot k_1 q \cdot k_3 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& -128 (p \cdot q)^2 q \cdot k_2 q \cdot k_3 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1} \\
& -128 p \cdot q p \cdot k_2 q \cdot k_2 q \cdot k_3 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1} \\
& +64 p \cdot q p \cdot k_2 (q \cdot k_2)^2 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1} \\
& +64 p \cdot q (q \cdot k_2)^2 q \cdot k_3 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1} \\
& +64 p \cdot k_2 (q \cdot k_1)^2 q \cdot k_3 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& -64 p \cdot k_2 q \cdot k_3 k_1 \cdot k_3 k_1 \cdot k_4 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& +64 p \cdot k_2 p \cdot k_4 q \cdot k_1 k_1 \cdot k_3 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& -32 p \cdot q p \cdot k_2 p \cdot k_4 k_1 \cdot k_3 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& +128 p \cdot k_2 q \cdot k_1 q \cdot k_3 k_1 \cdot k_3 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& -192 p \cdot q q \cdot k_1 q \cdot k_3 k_1 \cdot k_3 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1}
\end{aligned}$$

TABLE IV. (CONTINUED)

$$+256 m^2 P_1^{-2} Q_4^{-1}$$

$$-256 m^2 P_2^{-1} Q_4^{-2}$$

$$-192 m^2 P_1^{-1} Q_3^{-1} Q_4^{-1}$$

$$-320 m^2 P_1^{-1} Q_1^{-1} Q_4^{-1}$$

$$-128 m^2 p.k_4 P_1^{-1} P_2^{-1} Q_4^{-2}$$

$$-128 m^2 p.k_2 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1}$$

$$-128 m^2 p.q P_1^{-2} Q_2^{-1} Q_4^{-1}$$

$$-704 m^2 p.q P_1^{-1} P_2^{-1} Q_1^{-1} Q_4^{-1}$$

$$+64 m^2 p.q P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1}$$

$$+128 m^2 p.q P_1^{-1} P_2^{-1} Q_4^{-2}$$

$$-256 m^2 p.q P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1}$$

$$-128 m^2 p.k_3 P_1^{-1} P_2^{-1} Q_4^{-2}$$

$$-192 m^2 p.k_4 P_1^{-1} P_2^{-1} Q_1^{-1} Q_4^{-1}$$

$$-64 m^2 p.k_2 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1}$$

$$-128 m^2 q.k_3 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1}$$

TABLE IV (CONTINUED)

$$+128 m^2 q.k_2 P_1^{-2} Q_3^{-1} Q_4^{-1}$$

$$+128 m^2 q.k_3 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1}$$

$$+128 m^2 q.k_1 P_1^{-2} Q_2^{-1} Q_4^{-1}$$

$$+256 m^2 q.k_2 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1}$$

$$-128 m^2 [2 \ 4] P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1}$$

$$+128 m^2 k_1.k_4 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1}$$

$$+128 m^2 k_2.k_3 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1}$$

$$+128 m^2 k_1.k_3 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1}$$

$$+96 m^2 k_1.k_2 P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1}$$

$$+64 m^2 k_2.k_4 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1}$$

$$-832 m^2 P_1^{-1} P_4^{-1} [1 \ 2]^{-1}$$

$$+64 m^2 P_1^{-1} P_2^{-1} [1 \ 2]^{-1}$$

$$-128 m^2 P_1^{-2} [1 \ 2]^{-1}$$

$$+128 m^2 P_1^{-1} Q_4^{-1} [1 \ 2]^{-1}$$

$$-256 m^2 P_1^{-1} Q_1^{-1} [1 \ 2]^{-1}$$

TABLE IV (CONTINUED)

$$+64 m^2 P_2^{-1} Q_1^{-1} [1\ 2]^{-1}$$

$$-384 m^2 P_1^{-1} Q_4^{-1} [2\ 3]^{-1}$$

$$-64 m^2 P_1^{-1} Q_1^{-1} [2\ 3]^{-1}$$

$$-224 m^2 P_1^{-1} Q_4^{-1} [2\ 4]^{-1}$$

$$+128 m^2 Q_1^{-1} Q_4^{-1} [2\ 3]^{-1}$$

$$-80 m^2 p.q P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1}$$

$$+160 m^2 p.k_3 P_1^{-1} P_2^{-1} Q_4^{-1} [2\ 3]^{-1}$$

$$-192 m^2 p.k_4 P_1^{-1} P_2^{-1} Q_4^{-1} [1\ 2]^{-1}$$

$$-96 m^2 p.k_4 P_1^{-1} P_3^{-1} Q_1^{-1} [2\ 3]^{-1}$$

$$+192 m^2 p.q P_1^{-1} P_3^{-1} Q_1^{-1} [2\ 3]^{-1}$$

$$-128 m^2 p.k_3 P_1^{-1} P_2^{-1} Q_4^{-1} [1\ 2]^{-1}$$

$$+464 m^2 p.q P_1^{-1} P_2^{-1} Q_4^{-1} [2\ 4]^{-1}$$

$$+64 m^2 p.k_2 P_1^{-1} P_4^{-1} Q_1^{-1} [1\ 2]^{-1}$$

$$+256 m^2 p.k_4 P_1^{-1} P_2^{-1} Q_4^{-1} [2\ 3]^{-1}$$

$$-192 m^2 p.q P_1^{-1} P_2^{-1} Q_4^{-1} [1\ 2]^{-1}$$

TABLE IV (CONTINUED)

$$+288 m^2 p.k_2 P_1^{-1} P_3^{-1} Q_1^{-1} [2\ 3]^{-1}$$

$$+320 m^2 p.q P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1}$$

$$-64 m^2 p.k_4 P_1^{-1} P_2^{-1} Q_4^{-1} [2\ 4]^{-1}$$

$$-64 m^2 q.k_1 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1}$$

$$-64 m^2 q.k_2 P_1^{-1} P_2^{-1} Q_4^{-1} [1\ 2]^{-1}$$

$$-192 m^2 q.k_2 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1}$$

$$+128 m^2 q.k_1 P_1^{-1} P_2^{-1} Q_4^{-1} [2\ 3]^{-1}$$

$$+128 m^2 q.k_2 P_1^{-1} P_2^{-1} Q_1^{-1} [1\ 2]^{-1}$$

$$+320 m^2 q.k_2 P_1^{-1} P_2^{-1} Q_3^{-1} [2\ 4]^{-1}$$

$$+128 m^2 q.k_3 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1}$$

$$-128 m^2 q.k_2 P_1^{-2} Q_4^{-1} [1\ 2]^{-1}$$

$$-64 m^2 q.k_1 P_1^{-1} P_2^{-1} Q_4^{-1} [2\ 4]^{-1}$$

$$-64 m^2 q.k_3 P_1^{-1} P_2^{-1} Q_4^{-1} [2\ 4]^{-1}$$

$$+128 m^2 k_2.k_4 P_1^{-2} Q_4^{-1} [1\ 2]^{-1}$$

$$+64 m^2 [1\ 2] P_1^{-1} P_2^{-1} Q_4^{-1} [2\ 3]^{-1}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& +128 m^2 [1 \ 3] P_1^{-2} Q_4^{-1} [1 \ 2]^{-1} \\
& +32 m^2 k_1 \cdot k_4 P_1^{-1} P_4^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& -64 m^2 k_1 \cdot k_3 P_1^{-1} P_3^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& +128 m^2 [2 \ 3] P_1^{-1} P_3^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& +64 m^2 [2 \ 3] P_1^{-1} P_2^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& +64 m^2 k_2 \cdot k_4 P_1^{-1} P_3^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& -32 m^2 k_1 \cdot k_4 P_1^{-1} P_4^{-1} Q_1^{-1} [1 \ 2]^{-1} \\
& +128 m^2 p \cdot k_4 P_3^{-1} Q_4^{-2} [1 \ 2]^{-1} \\
& +640 m^2 p \cdot k_2 P_4^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& +288 m^2 p \cdot q P_1^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& +128 m^2 p \cdot k_4 P_1^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& +256 m^2 p \cdot k_4 P_2^{-1} Q_4^{-2} [1 \ 2]^{-1} \\
& +128 m^2 p \cdot k_2 P_1^{-1} Q_4^{-2} [1 \ 2]^{-1} \\
& +320 m^2 p \cdot k_4 P_1^{-1} Q_1^{-1} Q_4^{-1} [2 \ 4]^{-1} \\
& +192 m^2 p \cdot k_4 P_1^{-1} Q_1^{-1} Q_4^{-1} [2 \ 3]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$+240 m^2 p.q P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1}$$

$$+128 m^2 p.k_2 P_1^{-1} Q_1^{-1} Q_4^{-1} [2\ 3]^{-1}$$

$$-128 m^2 p.q P_3^{-1} Q_4^{-2} [1\ 2]^{-1}$$

$$+384 m^2 p.k_2 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1}$$

$$+192 m^2 p.q P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1}$$

$$+368 m^2 p.q P_1^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1}$$

$$-256 m^2 p.q P_2^{-1} Q_4^{-2} [1\ 2]^{-1}$$

$$+512 m^2 p.k_1 P_2^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1}$$

$$+192 m^2 p.k_4 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1}$$

$$-320 m^2 p.k_4 P_3^{-1} Q_1^{-1} Q_4^{-1} [2\ 3]^{-1}$$

$$-480 m^2 p.q P_3^{-1} Q_1^{-1} Q_4^{-1} [2\ 3]^{-1}$$

$$+256 m^2 p.k_1 P_2^{-1} Q_3^{-1} Q_4^{-1} [2\ 4]^{-1}$$

$$-448 m^2 p.k_3 P_1^{-1} Q_3^{-1} Q_4^{-1} [2\ 4]^{-1}$$

$$+64 m^2 p.k_3 P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1}$$

$$+320 m^2 q.k_3 P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& -64 m^2 q.k_2 P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +576 m^2 q.k_3 P_2^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& -384 m^2 q.k_2 P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +288 m^2 q.k_2 P_1^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +96 m^2 q.k_3 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +224 m^2 q.k_3 P_3^{-1} Q_1^{-1} Q_4^{-1} [2\ 3]^{-1} \\
& +32 m^2 q.k_2 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& -256 m^2 q.k_3 P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +128 m^2 k_1.k_3 P_1^{-1} Q_4^{-2} [1\ 2]^{-1} \\
& -192 m^2 [2\ 4] P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& -128 m^2 [1\ 2] P_1^{-1} Q_4^{-2} [2\ 3]^{-1} \\
& +64 m^2 [1\ 2] P_1^{-1} Q_1^{-1} Q_4^{-1} [2\ 3]^{-1} \\
& -64 m^2 [2\ 4] P_2^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +64 m^2 k_1.k_3 P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& -128 m^2 [2\ 4] P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& -192 m^2 [1 \ 4] P_1^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& +64 m^2 [1 \ 2] P_1^{-1} Q_3^{-1} Q_4^{-1} [2 \ 4]^{-1} \\
& -192 m^2 [2 \ 4] P_1^{-1} Q_3^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& -64 m^2 k_1 \cdot k_3 P_3^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& +128 m^2 p \cdot k_2 p \cdot k_4 P_1^{-1} P_3^{-1} Q_4^{-2} [1 \ 2]^{-1} \\
& +384 m^2 (p \cdot k_2)^2 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& +512 m^2 (p \cdot q)^2 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& -64 m^2 p \cdot q p \cdot k_4 P_1^{-1} P_2^{-1} Q_1^{-1} Q_4^{-1} [2 \ 4]^{-1} \\
& -384 m^2 (p \cdot q)^2 P_1^{-1} P_2^{-1} Q_1^{-1} Q_4^{-1} [2 \ 4]^{-1} \\
& -192 m^2 (p \cdot q)^2 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [2 \ 3]^{-1} \\
& +352 m^2 (p \cdot q)^2 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& +192 m^2 (p \cdot q)^2 P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [2 \ 4]^{-1} \\
& -64 m^2 p \cdot k_2 p \cdot k_4 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& -128 m^2 p \cdot q p \cdot k_2 P_1^{-1} P_3^{-1} Q_4^{-2} [1 \ 2]^{-1} \\
& +448 m^2 p \cdot q p \cdot k_2 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& -128 m^2 (p.q)^2 P_1^{-2} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& -128 m^2 (p.q)^2 P_1^{-1} P_2^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +64 m^2 p.q p.k_2 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [2\ 3]^{-1} \\
& +96 m^2 p.q p.k_2 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& -64 m^2 p.k_3 p.k_4 P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [2\ 4]^{-1} \\
& -128 m^2 p.q p.k_3 P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [2\ 4]^{-1} \\
& -96 m^2 p.q p.k_2 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +256 m^2 p.q q.k_1 P_1^{-2} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& -64 m^2 p.q q.k_2 P_1^{-1} P_2^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +256 m^2 p.k_4 q.k_2 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +64 m^2 p.k_4 q.k_3 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +128 m^2 p.q q.k_1 P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [2\ 4]^{-1} \\
& -224 m^2 p.k_2 q.k_3 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +320 m^2 p.k_2 q.k_1 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +512 m^2 p.k_2 q.k_3 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& -384 m^2 p \cdot q \, q \cdot k_2 \, P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1 \, 2]^{-1} \\
& +192 m^2 p \cdot k_4 \, q \cdot k_3 \, P_1^{-1} P_2^{-1} Q_1^{-1} Q_4^{-1} [2 \, 4]^{-1} \\
& +192 m^2 p \cdot q \, q \cdot k_2 \, P_1^{-1} P_2^{-1} Q_1^{-1} Q_4^{-1} [2 \, 4]^{-1} \\
& -160 m^2 p \cdot k_2 \, q \cdot k_2 \, P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1 \, 2]^{-1} \\
& +32 m^2 p \cdot q \, q \cdot k_2 \, P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1 \, 2]^{-1} \\
& +256 m^2 p \cdot k_3 \, q \cdot k_2 \, P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [2 \, 4]^{-1} \\
& -384 m^2 p \cdot q \, q \cdot k_1 \, P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [1 \, 2]^{-1} \\
& -224 m^2 p \cdot q \, q \cdot k_3 \, P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1 \, 2]^{-1} \\
& -416 m^2 p \cdot q \, q \cdot k_1 \, P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1 \, 2]^{-1} \\
& +320 m^2 p \cdot q \, q \cdot k_3 \, P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1 \, 2]^{-1} \\
& -128 m^2 p \cdot k_2 \, q \cdot k_2 \, P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1 \, 2]^{-1} \\
& -128 m^2 (q \cdot k_1)^2 P_1^{-2} Q_2^{-1} Q_4^{-1} [1 \, 2]^{-1} \\
& -128 m^2 q \cdot k_1 \, q \cdot k_2 \, P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [1 \, 2]^{-1} \\
& +192 m^2 (q \cdot k_3)^2 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1 \, 2]^{-1} \\
& +256 m^2 (q \cdot k_3)^2 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1 \, 2]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& +128 m^2 (q.k_2)^2 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& -128 m^2 (q.k_1)^2 P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +64 m^2 q.k_1 q.k_2 P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [2\ 4]^{-1} \\
& +128 m^2 (q.k_1)^2 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& -128 m^2 p.k_3 [1\ 2] P_1^{-1} P_2^{-1} Q_4^{-2} [2\ 3]^{-1} \\
& -128 m^2 p.k_4 [1\ 2] P_1^{-1} P_2^{-1} Q_1^{-1} Q_4^{-1} [2\ 4]^{-1} \\
& -128 m^2 p.q [1\ 3] P_1^{-2} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& -192 m^2 p.q [1\ 2] P_1^{-1} P_2^{-1} Q_1^{-1} Q_4^{-1} [2\ 3]^{-1} \\
& +64 m^2 p.k_4 [1\ 2] P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [2\ 4]^{-1} \\
& -128 m^2 p.q k_1.k_4 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& -32 m^2 p.k_2 k_1.k_4 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +64 m^2 p.q k_1.k_4 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& -160 m^2 p.q k_1.k_3 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& -128 m^2 p.k_2 [2\ 4] P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& -64 m^2 p.q [2\ 4] P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& -192 m^2 p.q [1\ 2] P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [2\ 3]^{-1} \\
& -128 m^2 p.q [2\ 3] P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +64 m^2 p.k_3 [1\ 2] P_1^{-1} P_2^{-1} Q_1^{-1} Q_4^{-1} [2\ 3]^{-1} \\
& -128 m^2 p.q [2\ 3] P_1^{-1} P_2^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& -192 m^2 p.q [2\ 4] P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +64 m^2 p.k_4 k_1.k_3 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +32 m^2 p.k_3 k_1.k_4 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& -64 m^2 p.k_2 k_1.k_3 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& -128 m^2 q.k_3 [2\ 4] P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +192 m^2 q.k_2 [1\ 2] P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [2\ 3]^{-1} \\
& +128 m^2 q.k_2 [2\ 3] P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& -64 m^2 q.k_2 [2\ 3] P_1^{-1} P_2^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +192 m^2 q.k_1 [2\ 4] P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +64 m^2 q.k_3 k_1.k_3 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& -64 m^2 q.k_2 k_1.k_4 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +192 m^2 q.k_3 [1\ 2] P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [2\ 3]^{-1} \\
& +128 m^2 q.k_1 [1\ 3] P_1^{-2} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& -64 m^2 q.k_2 [1\ 2] P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [2\ 4]^{-1} \\
& +192 m^2 q.k_2 [2\ 4] P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& -64 m^2 k_1.k_3 k_1.k_4 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +64 m^2 q.k_1 k_1.k_3 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& -192 m^2 P_3^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -160 m^2 P_1^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +1056 m^2 P_1^{-1} [1\ 2]^{-1} [1\ 3]^{-1} \\
& +896 m^2 Q_4^{-1} [1\ 2]^{-1} [1\ 3]^{-1} \\
& +544 m^2 Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +1088 m^2 Q_4^{-1} [1\ 2]^{-1} [1\ 4]^{-1} \\
& +256 m^2 p.q P_1^{-1} P_3^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& +128 m^2 p.q P_1^{-2} [1\ 2]^{-1} [1\ 3]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$+256 m^2 p.k_2 P_1^{-1} P_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$+192 m^2 p.k_4 P_1^{-1} P_2^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$+64 m^2 p.q P_1^{-1} P_2^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$+64 m^2 p.q P_1^{-1} P_3^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+320 m^2 q.k_3 P_1^{-1} P_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$-64 m^2 q.k_2 P_1^{-1} P_2^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-128 m^2 q.k_1 P_1^{-2} [1\ 2]^{-1} [1\ 3]^{-1}$$

$$-64 m^2 q.k_4 P_1^{-1} P_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-64 m^2 k_1.k_4 P_1^{-1} P_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+256 m^2 p.k_2 P_1^{-1} Q_4^{-1} [1\ 2]^{-2}$$

$$+672 m^2 p.q P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-448 m^2 p.k_2 P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$+160 m^2 p.k_2 P_4^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$-64 m^2 p.q P_1^{-1} Q_2^{-1} [1\ 2]^{-1} [1\ 4]^{-1}$$

$$-352 m^2 p.q P_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

TABLE IV (CONTINUED)

$$+224 m^2 p.k_2 P_1^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-64 m^2 p.k_3 P_2^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$-112 m^2 p.q P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+512 m^2 p.k_4 P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+480 m^2 p.k_4 P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-192 m^2 p.k_1 P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$+64 m^2 p.k_1 P_4^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$+160 m^2 p.q P_1^{-1} Q_4^{-1} [1\ 2]^{-1} [1\ 3]^{-1}$$

$$+64 m^2 p.k_3 P_2^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$+800 m^2 p.k_4 P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-448 m^2 p.q P_1^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-64 m^2 p.k_1 P_2^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$+64 m^2 p.k_1 P_2^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$+32 m^2 p.q P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+352 m^2 p.q P_1^{-1} Q_1^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

TABLE IV (CONTINUED)

$$+224 m^2 p.k_2 P_1^{-1} Q_1^{-1} [1 2]^{-1} [3 4]^{-1}$$

$$+140 m^2 p.k_4 P_1^{-1} Q_1^{-1} [1 2]^{-1} [3 4]^{-1}$$

$$+572 m^2 p.k_2 P_4^{-1} Q_1^{-1} [1 2]^{-1} [3 4]^{-1}$$

$$-416 m^2 p.k_4 P_3^{-1} Q_4^{-1} [1 2]^{-1} [3 4]^{-1}$$

$$+832 m^2 p.k_1 P_4^{-1} Q_1^{-1} [1 2]^{-1} [3 4]^{-1}$$

$$-128 m^2 p.k_1 P_3^{-1} Q_4^{-1} [1 2]^{-1} [3 4]^{-1}$$

$$+256 m^2 q.k_3 P_1^{-1} Q_4^{-1} [1 2]^{-2}$$

$$+544 m^2 q.k_1 P_1^{-1} Q_2^{-1} [1 2]^{-1} [1 4]^{-1}$$

$$+128 m^2 q.k_2 P_1^{-1} Q_4^{-1} [1 2]^{-1} [2 4]^{-1}$$

$$+64 m^2 q.k_3 P_1^{-1} Q_1^{-1} [1 2]^{-1} [2 3]^{-1}$$

$$-64 m^2 q.k_4 P_1^{-1} Q_2^{-1} [1 2]^{-1} [1 3]^{-1}$$

$$+64 m^2 q.k_4 P_1^{-1} Q_1^{-1} [1 2]^{-1} [2 3]^{-1}$$

$$+576 m^2 q.k_1 P_2^{-1} Q_3^{-1} [1 2]^{-1} [2 4]^{-1}$$

$$-192 m^2 q.k_4 P_1^{-1} Q_3^{-1} [1 2]^{-1} [2 4]^{-1}$$

$$-128 m^2 q.k_2 P_1^{-1} Q_4^{-1} [1 2]^{-1} [3 4]^{-1}$$

TABLE IV (CONTINUED)

$$-608 m^2 q.k_2 P_4^{-1} Q_1^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-32 m^2 q.k_3 P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-704 m^2 q.k_2 P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-768 m^2 q.k_1 P_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$+96 m^2 q.k_2 P_1^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-448 m^2 q.k_3 P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$+288 m^2 q.k_2 P_2^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-128 m^2 q.k_1 P_2^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$-640 m^2 q.k_3 P_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$+576 m^2 q.k_3 P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-128 m^2 q.k_4 P_1^{-1} Q_1^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-128 m^2 q.k_2 P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+32 m^2 k_1.k_4 P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+192 m^2 k_1.k_3 P_3^{-1} Q_2^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-96 m^2 k_2.k_4 P_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

TABLE IV (CONTINUED)

$$-128 m^2 p.k_4 Q_4^{-2} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-64 m^2 p.q Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [1\ 4]^{-1}$$

$$+192 m^2 p.k_4 Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-128 m^2 p.k_1 Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+128 m^2 p.q Q_4^{-2} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-64 m^2 p.q Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$+16 m^2 p.q Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+64 m^2 q.k_2 Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$+320 m^2 q.k_2 Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-192 m^2 q.k_3 Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [1\ 4]^{-1}$$

$$+256 m^2 q.k_1 Q_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$-64 m^2 k_1.k_3 Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+32 m^2 k_1.k_4 Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-128 m^2 k_1.k_3 Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-256 m^2 (p.q)^2 P_1^{-2} Q_2^{-1} [1\ 2]^{-1} [1\ 4]^{-1}$$

TABLE IV (CONTINUED)

$$+448 m^2 p.q p.k_4 P_1^{-1} P_2^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$-256 m^2 (p.q)^2 P_1^{-1} P_2^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$+96 m^2 p.k_2 p.k_4 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-416 m^2 p.q p.k_2 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$+256 m^2 p.q p.k_3 P_1^{-1} P_2^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$+128 m^2 p.q p.k_3 P_1^{-1} P_2^{-1} Q_3^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$-32 m^2 p.k_2 p.k_4 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+224 m^2 p.q p.k_2 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+64 m^2 (p.k_2)^2 P_1^{-1} P_4^{-1} Q_1^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-448 m^2 (p.q)^2 P_1^{-1} P_4^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$-256 m^2 (p.q)^2 P_1^{-1} P_2^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$+32 m^2 (p.k_2)^2 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$+256 m^2 p.q p.k_4 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-256 m^2 (p.q)^2 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-128 m^2 (p.q)^2 P_1^{-1} P_2^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& -224 m^2 (p.k_4)^2 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +64 m^2 p.q\ p.k_2 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -64 m^2 p.k_2\ p.k_3 P_1^{-1} P_4^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +160 m^2 p.q\ p.k_2 P_1^{-1} P_4^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +368 m^2 (p.q)^2 P_1^{-1} P_4^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +368 m^2 p.q\ p.k_2 P_1^{-1} P_3^{-1} Q_2^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -64 m^2 (p.q)^2 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +256 m^2 p.q\ p.k_2 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -64 m^2 p.k_2\ p.k_4 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -320 m^2 (p.k_2)^2 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& +192 m^2 p.k_2\ q.k_1 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& +96 m^2 p.q\ q.k_2 P_1^{-1} P_4^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +512 m^2 p.q\ q.k_1 P_1^{-2} Q_2^{-1} [1\ 2]^{-1} [1\ 4]^{-1} \\
& +288 m^2 p.q\ q.k_2 P_1^{-1} P_2^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& -128 m^2 p.k_4\ q.k_2 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& +64 m^2 p.k_2 q.k_3 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -64 m^2 p.k_4 q.k_4 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& +64 m^2 p.q q.k_3 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& +128 m^2 p.q q.k_3 P_1^{-1} P_2^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& +64 m^2 p.q q.k_2 P_1^{-1} P_2^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -192 m^2 p.k_4 q.k_1 P_1^{-1} P_2^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +32 m^2 p.k_4 q.k_2 P_1^{-1} P_2^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +256 m^2 p.q q.k_2 P_1^{-1} P_2^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +160 m^2 p.k_2 q.k_2 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& +32 m^2 p.k_4 q.k_2 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& +128 m^2 p.q q.k_2 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& +128 m^2 p.q q.k_3 P_1^{-2} Q_2^{-1} [1\ 2]^{-1} [1\ 3]^{-1} \\
& -192 m^2 p.k_3 q.k_2 P_1^{-1} P_2^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& +128 m^2 p.q q.k_1 P_1^{-1} P_2^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +96 m^2 p.k_3 q.k_2 P_1^{-1} P_2^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& -192 m^2 p.q q.k_1 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +192 m^2 p.k_2 q.k_3 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +32 m^2 p.k_4 q.k_2 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -48 m^2 p.q q.k_3 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -256 m^2 p.q q.k_4 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -224 m^2 p.k_2 q.k_1 P_1^{-1} P_3^{-1} Q_2^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -64 m^2 p.k_3 q.k_2 P_1^{-1} P_2^{-1} Q_3^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& -256 m^2 p.k_2 q.k_3 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -128 m^2 p.k_4 q.k_2 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -128 m^2 p.k_4 q.k_4 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -1152 m^2 p.q q.k_3 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +96 m^2 p.k_2 q.k_1 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -128 m^2 p.k_2 q.k_3 P_1^{-1} P_3^{-1} Q_2^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -256 m^2 (q.k_1)^2 P_1^{-2} Q_2^{-1} [1\ 2]^{-1} [1\ 4]^{-1} \\
& +64 m^2 q.k_2 q.k_4 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 3]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& -64 m^2 q.k_2 q.k_4 P_1^{-1} P_2^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -128 m^2 q.k_2 q.k_3 P_1^{-1} P_2^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +64 m^2 (q.k_3)^2 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +128 m^2 q.k_3 q.k_4 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +128 m^2 q.k_1 q.k_3 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -128 m^2 (q.k_2)^2 P_1^{-1} P_2^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& -128 m^2 q.k_1 q.k_3 P_1^{-2} Q_2^{-1} [1\ 2]^{-1} [1\ 3]^{-1} \\
& -64 m^2 q.k_1 q.k_2 P_1^{-1} P_2^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +64 m^2 q.k_2 q.k_3 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +320 m^2 (q.k_3)^2 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +64 m^2 q.k_1 q.k_2 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +64 m^2 q.k_3 k_1.k_3 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +32 m^2 q.k_2 k_1.k_3 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -64 m^2 k_1.k_3 k_2.k_3 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -64 m^2 p.q k_1.k_4 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [3\ 4]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& -32 m^2 p.k_2 k_1.k_4 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -128 m^2 p.k_4 k_2.k_4 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -32 m^2 p.k_2 k_1.k_4 P_1^{-1} P_4^{-1} Q_1^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +128 m^2 p.q k_1.k_4 P_1^{-1} P_4^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +224 m^2 p.k_2 k_1.k_3 P_1^{-1} P_3^{-1} Q_2^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -208 m^2 p.q k_1.k_3 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +128 m^2 (p.q)^2 P_1^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [1\ 4]^{-1} \\
& -128 m^2 p.k_3 p.k_4 P_1^{-1} Q_4^{-2} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -192 m^2 p.k_2 p.k_4 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& -64 m^2 p.q p.k_4 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& -64 m^2 p.k_2 p.k_4 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& +384 m^2 p.q p.k_2 P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& +288 m^2 (p.q)^2 P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& +128 m^2 p.q p.k_4 P_2^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -64 m^2 p.q p.k_2 P_1^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [1\ 4]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& +128 m^2 p.q p.k_3 P_1^{-1} Q_4^{-2} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -320 m^2 (p.k_2)^2 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +128 m^2 p.q p.k_1 P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -64 m^2 p.q p.k_2 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -64 m^2 (p.q)^2 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& +192 m^2 p.q p.k_4 P_2^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -128 m^2 p.q p.k_4 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -192 m^2 (p.q)^2 P_2^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& +256 m^2 p.q p.k_1 P_2^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +128 m^2 (p.k_4)^2 P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -128 m^2 (p.k_3)^2 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -96 m^2 p.q p.k_4 P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -256 m^2 p.q p.k_4 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -496 m^2 (p.q)^2 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +64 m^2 p.k_1 p.k_3 P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& -64 m^2 p.k_1 p.k_4 P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -304 m^2 p.q p.k_1 P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -256 m^2 p.k_2 p.k_4 P_3^{-1} Q_4^{-2} [1\ 2]^{-1} [2\ 3]^{-1} \\
& +256 m^2 (p.q)^2 P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& -320 m^2 p.q p.k_2 P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +64 m^2 p.k_3 p.k_4 P_1^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +96 m^2 p.q p.k_3 P_1^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +128 m^2 (p.k_2)^2 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -192 m^2 p.k_3 p.k_4 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +416 m^2 p.q p.k_3 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +368 m^2 (p.q)^2 P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -128 m^2 p.k_2 p.k_3 P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +192 m^2 p.q p.k_2 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +64 m^2 p.k_2 p.k_4 P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -560 m^2 p.q p.k_2 P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& -256 m^2 p.q q.k_1 P_1^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [1\ 4]^{-1} \\
& -192 m^2 p.k_4 q.k_3 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& -192 m^2 p.k_2 q.k_2 P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& +192 m^2 p.q q.k_3 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& +128 m^2 p.k_3 q.k_1 P_1^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [1\ 3]^{-1} \\
& -64 m^2 p.k_1 q.k_2 P_2^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& +320 m^2 p.q q.k_2 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -64 m^2 p.k_2 q.k_1 P_1^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +128 m^2 p.k_4 q.k_2 P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +64 m^2 p.k_2 q.k_1 P_1^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [1\ 4]^{-1} \\
& -128 m^2 p.q q.k_3 P_1^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [1\ 4]^{-1} \\
& -32 m^2 p.k_4 q.k_3 P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -160 m^2 p.q q.k_2 P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -64 m^2 p.q q.k_1 P_1^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [1\ 3]^{-1} \\
& +128 m^2 p.q q.k_3 P_2^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$+32 m^2 p.k_3 q.k_2 P_1^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$+32 m^2 p.q q.k_1 P_1^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$-576 m^2 p.k_4 q.k_3 P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+416 m^2 p.k_2 q.k_3 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+352 m^2 p.k_4 q.k_3 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+64 m^2 p.q q.k_2 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-384 m^2 p.k_1 q.k_3 P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-384 m^2 p.k_2 q.k_3 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$-64 m^2 p.q q.k_2 P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$-256 m^2 p.q q.k_3 P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$+64 m^2 p.k_2 q.k_2 P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$-352 m^2 p.k_3 q.k_2 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+592 m^2 p.q q.k_3 P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+672 m^2 p.q q.k_3 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+640 m^2 p.k_2 q.k_1 P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& -192 m^2 (q.k_3)^2 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& -192 m^2 (q.k_3)^2 P_1^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [1\ 4]^{-1} \\
& +64 m^2 q.k_1 q.k_3 P_1^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [1\ 4]^{-1} \\
& +128 m^2 (q.k_1)^2 P_1^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [1\ 3]^{-1} \\
& +544 m^2 (q.k_3)^2 P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +128 m^2 (q.k_1)^2 P_1^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [1\ 4]^{-1} \\
& -192 m^2 (q.k_3)^2 P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& +192 m^2 q.k_1 q.k_2 P_2^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +128 m^2 q.k_2 q.k_3 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -192 m^2 p.k_4 k_1.k_3 P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +32 m^2 p.k_2 k_1.k_4 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +320 m^2 p.q k_1.k_4 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +64 m^2 q.k_3 k_1.k_4 P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -64 m^2 p.k_4 k_1.k_3 P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -32 m^2 q.k_1 k_1.k_3 P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$+80 m^2 p \cdot q k_1 \cdot k_3 P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+128 m^2 (p \cdot q)^3 P_1^{-2} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [1\ 4]^{-1}$$

$$+64 m^2 p \cdot q (p \cdot k_2)^2 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-64 m^2 (p \cdot k_2)^2 p \cdot k_4 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+64 m^2 (p \cdot k_4)^3 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+576 m^2 (p \cdot q)^2 p \cdot k_4 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-64 m^2 p \cdot q (p \cdot k_2)^2 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-128 m^2 (p \cdot q)^3 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-48 m^2 p \cdot q p \cdot k_2 p \cdot k_4 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+256 m^2 (p \cdot q)^3 P_1^{-1} P_2^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$+320 m^2 (p \cdot q)^2 p \cdot k_2 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-256 m^2 p \cdot q (p \cdot k_4)^2 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-672 m^2 (p \cdot q)^3 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-432 m^2 (p \cdot q)^2 p \cdot k_2 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-64 m^2 p \cdot q (p \cdot k_2)^2 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

TABLE IV (CONTINUED)

$$+96 m^2 (p.q)^3 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+128 m^2 (p.q)^2 p.k_2 P_1^{-1} P_3^{-1} Q_4^{-2} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-128 m^2 p.q p.k_2 p.k_4 P_1^{-1} P_3^{-1} Q_4^{-2} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-128 m^2 (p.k_2)^2 p.k_4 P_1^{-1} P_3^{-1} Q_4^{-2} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-128 m^2 (p.k_2)^3 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$+128 m^2 (p.q)^3 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$-128 m^2 (p.q)^2 p.k_2 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$-192 m^2 p.q (p.k_2)^2 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$-512 m^2 (p.q)^2 q.k_2 P_1^{-1} P_2^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$+128 m^2 p.q p.k_4 q.k_2 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$+128 m^2 (p.k_3)^2 q.k_1 P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$+128 m^2 p.q p.k_4 q.k_1 P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$-320 m^2 p.q p.k_3 q.k_1 P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$-512 m^2 p.q p.k_4 q.k_2 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+800 m^2 (p.q)^2 q.k_3 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& +96 m^2 p \cdot q p \cdot k_2 q \cdot k_1 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& -384 m^2 (p \cdot q)^2 q \cdot k_1 P_1^{-2} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1} [1 \ 4]^{-1} \\
& -128 m^2 (p \cdot q)^2 q \cdot k_2 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1} \\
& -128 m^2 p \cdot q p \cdot k_2 q \cdot k_3 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1} \\
& +64 m^2 (p \cdot k_4)^2 q \cdot k_1 P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1} \\
& +192 m^2 (p \cdot q)^2 q \cdot k_1 P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1} \\
& +512 m^2 p \cdot q p \cdot k_2 q \cdot k_3 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& -32 m^2 (p \cdot q)^2 q \cdot k_2 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& -16 m^2 (p \cdot q)^2 q \cdot k_2 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& +288 m^2 p \cdot q p \cdot k_2 q \cdot k_3 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& -256 m^2 (p \cdot k_2)^2 q \cdot k_3 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1} \\
& -256 m^2 p \cdot q p \cdot k_2 q \cdot k_3 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1} \\
& -128 m^2 (p \cdot q)^2 q \cdot k_2 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1} \\
& -256 m^2 (p \cdot q)^2 q \cdot k_3 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1} \\
& +192 m^2 p \cdot q p \cdot k_2 q \cdot k_2 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& +64 m^2 (p.k_2)^2 q.k_2 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +384 m^2 p.q (q.k_1)^2 P_1^{-2} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [1\ 4]^{-1} \\
& +256 m^2 p.q (q.k_2)^2 P_1^{-1} P_2^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +192 m^2 p.k_3 q.k_1 q.k_2 P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& -832 m^2 p.q (q.k_3)^2 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -32 m^2 p.k_2 (q.k_1)^2 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +128 m^2 p.q (q.k_1)^2 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -192 m^2 p.k_2 (q.k_3)^2 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& -64 m^2 p.q (q.k_3)^2 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +192 m^2 p.q q.k_2 q.k_3 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& -64 m^2 p.k_2 (q.k_2)^2 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +64 m^2 p.k_2 q.k_2 q.k_3 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +192 m^2 p.q q.k_2 q.k_3 P_1^{-1} P_2^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& -192 m^2 p.q q.k_1 q.k_2 P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& -128 m^2 (q.k_1)^3 P_1^{-2} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [1\ 4]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& -64 m^2 (q.k_1)^3 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -64 m^2 (q.k_3)^3 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& -64 m^2 (q.k_2)^2 q.k_3 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +192 m^2 (q.k_3)^3 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +64 m^2 p.k_4 (k_1.k_4)^2 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +64 m^2 q.k_3 k_1.k_4 k_2.k_3 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -128 m^2 p.k_2 p.k_4 k_1.k_3 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +64 m^2 q.k_2 q.k_3 k_1.k_3 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -128 m^2 p.q k_1.k_3 k_2.k_4 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -64 m^2 p.q q.k_2 k_1.k_3 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +64 m^2 p.q p.k_2 k_1.k_4 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -128 m^2 p.q p.k_4 k_1.k_3 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +128 m^2 (p.q)^2 k_1.k_3 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -64 m^2 p.k_2 (k_1.k_4)^2 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -32 m^2 p.q (k_1.k_4)^2 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$+32 m^2 p.k_2 p.k_3 k_1.k_4 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-32 m^2 p.k_2 q.k_3 k_1.k_4 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-64 m^2 (p.q)^2 k_1.k_4 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-32 m^2 p.k_2 q.k_1 k_1.k_3 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+16 m^2 p.q p.k_2 k_1.k_3 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-112 m^2 p.q q.k_1 k_1.k_3 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+64 m^2 (p.q)^2 k_1.k_3 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-128 m^4 P_1^{-1} P_2^{-1} Q_4^{-2}$$

$$-256 m^4 P_1^{-1} P_2^{-1} Q_1^{-1} Q_4^{-1}$$

$$-352 m^4 P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1}$$

$$-320 m^4 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1}$$

$$+128 m^4 P_1^{-2} Q_2^{-1} Q_4^{-1}$$

$$-384 m^4 P_1^{-2} Q_4^{-1} [1\ 2]^{-1}$$

$$-368 m^4 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1}$$

$$-160 m^4 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& -208 m^4 P_1^{-1} P_2^{-1} Q_4^{-1} [2 \ 4]^{-1} \\
& +128 m^4 P_1^{-1} P_2^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& +96 m^4 P_1^{-1} P_3^{-1} Q_1^{-1} [2 \ 3]^{-1} \\
& -384 m^4 P_1^{-1} Q_4^{-2} [1 \ 2]^{-1} \\
& +608 m^4 P_1^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& +544 m^4 P_1^{-1} Q_3^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& -128 m^4 P_1^{-1} Q_4^{-2} [2 \ 3]^{-1} \\
& +64 m^4 P_1^{-1} Q_1^{-1} Q_4^{-1} [2 \ 3]^{-1} \\
& +448 m^4 P_4^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& +912 m^4 P_1^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& -304 m^4 P_3^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& +128 m^4 p.k_2 P_1^{-1} P_3^{-1} Q_4^{-2} [1 \ 2]^{-1} \\
& +384 m^4 p.q P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& -64 m^4 p.q P_1^{-1} P_2^{-1} Q_4^{-2} [1 \ 2]^{-1} \\
& +384 m^4 p.k_4 P_1^{-1} P_2^{-1} Q_1^{-1} Q_4^{-1} [2 \ 4]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& +128 m^4 p.q P_1^{-1} P_2^{-1} Q_1^{-1} Q_4^{-1} [2\ 4]^{-1} \\
& +64 m^4 p.q P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [2\ 3]^{-1} \\
& +1024 m^4 p.q P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +64 m^4 p.q P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& -64 m^4 p.q P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +768 m^4 p.k_2 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +64 m^4 p.k_4 P_1^{-1} P_2^{-1} Q_4^{-2} [1\ 2]^{-1} \\
& +192 m^4 p.k_3 P_1^{-1} P_2^{-1} Q_4^{-2} [1\ 2]^{-1} \\
& +256 m^4 p.k_2 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [2\ 3]^{-1} \\
& -32 m^4 p.k_2 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& -64 m^4 p.k_3 P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [2\ 4]^{-1} \\
& +608 m^4 p.k_2 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +128 m^4 q.k_2 P_1^{-2} Q_4^{-2} [1\ 2]^{-1} \\
& +128 m^4 q.k_2 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +192 m^4 q.k_3 P_1^{-1} P_2^{-1} Q_1^{-1} Q_4^{-1} [2\ 4]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& -352 m^4 q.k_2 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +64 m^4 q.k_1 P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [2\ 4]^{-1} \\
& -128 m^4 q.k_1 P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +768 m^4 q.k_3 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +256 m^4 q.k_3 P_1^{-2} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +256 m^4 q.k_3 P_1^{-1} P_2^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& +128 m^4 q.k_3 P_1^{-2} Q_2^{-1} Q_4^{-1} [1\ 3]^{-1} \\
& -288 m^4 q.k_3 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& -96 m^4 q.k_1 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& -128 m^4 k_2.k_4 P_1^{-2} Q_4^{-2} [1\ 2]^{-1} \\
& -256 m^4 [2\ 4] P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} \\
& -128 m^4 [1\ 2] P_1^{-1} P_2^{-1} Q_4^{-2} [2\ 3]^{-1} \\
& -64 m^4 [1\ 2] P_1^{-1} P_2^{-1} Q_1^{-1} Q_4^{-1} [2\ 4]^{-1} \\
& +64 m^4 [1\ 2] P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [2\ 3]^{-1} \\
& -128 m^4 [1\ 2] P_1^{-2} Q_2^{-1} Q_4^{-1} [1\ 3]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& +64 m^4 [1 \ 2] P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [2 \ 4]^{-1} \\
& -192 m^4 k_1 \cdot k_4 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& -128 m^4 [2 \ 3] P_1^{-1} P_3^{-1} Q_4^{-2} [1 \ 2]^{-1} \\
& -128 m^4 [1 \ 4] P_1^{-2} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& -128 m^4 [2 \ 3] P_1^{-1} P_2^{-1} Q_4^{-2} [1 \ 2]^{-1} \\
& -128 m^4 [2 \ 4] P_1^{-1} P_2^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& +64 m^4 [2 \ 3] P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& +64 m^4 [1 \ 2] P_1^{-1} P_2^{-1} Q_1^{-1} Q_4^{-1} [2 \ 3]^{-1} \\
& -192 m^4 [2 \ 4] P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& -32 m^4 k_1 \cdot k_3 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1 \ 2]^{-1} \\
& +320 m^4 P_1^{-1} P_3^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1} \\
& +832 m^4 P_1^{-1} P_2^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1} \\
& +256 m^4 P_1^{-2} [1 \ 2]^{-1} [1 \ 3]^{-1} \\
& -192 m^4 P_1^{-1} P_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& -512 m^4 P_1^{-1} Q_4^{-1} [1 \ 2]^{-2}
\end{aligned}$$

TABLE IV' (CONTINUED)

$$+96 m^4 P_1^{-1} Q_2^{-1} [1\ 2]^{-1} [1\ 4]^{-1}$$

$$-608 m^4 P_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$+192 m^4 P_1^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$+384 m^4 P_1^{-1} Q_1^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+416 m^4 P_1^{-1} Q_4^{-1} [1\ 2]^{-1} [1\ 3]^{-1}$$

$$-768 m^4 P_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$+1056 m^4 P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-128 m^4 Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$+704 m^4 Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$+352 m^4 Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+256 m^4 p.q P_1^{-1} P_2^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-352 m^4 p.k_4 P_1^{-1} P_2^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$-320 m^4 p.q P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-32 m^4 p.q P_1^{-1} P_2^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$-128 m^4 p.k_4 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& +288 m^4 p.q P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -160 m^4 p.k_2 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -576 m^4 p.k_2 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -256 m^4 p.k_4 P_1^{-1} P_2^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -640 m^4 p.k_3 P_1^{-1} P_2^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -352 m^4 p.k_2 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& +32 m^4 p.k_3 P_1^{-1} P_2^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& -304 m^4 p.k_4 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -160 m^4 p.q P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +320 m^4 p.k_3 P_1^{-1} P_4^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +128 m^4 q.k_3 P_1^{-2} Q_4^{-1} [1\ 2]^{-2} \\
& +384 m^4 q.k_1 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& +416 m^4 q.k_2 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& +128 m^4 q.k_3 P_1^{-2} Q_4^{-1} [1\ 2]^{-1} [1\ 3]^{-1} \\
& +64 m^4 q.k_2 P_1^{-1} P_2^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$+544 m^4 q.k_2 P_1^{-1} P_2^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$+128 m^4 q.k_3 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+128 m^4 q.k_3 P_1^{-1} P_2^{-1} Q_4^{-1} [1\ 2]^{-2}$$

$$-64 m^4 q.k_4 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$+256 m^4 q.k_3 P_1^{-1} P_2^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$+256 m^4 q.k_1 P_1^{-1} P_2^{-1} Q_3^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$-400 m^4 q.k_3 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+32 m^4 q.k_2 P_1^{-1} P_3^{-1} Q_1^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-64 m^4 k_2.k_4 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-128 m^4 k_1.k_4 P_1^{-1} P_4^{-1} Q_1^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+208 m^4 k_1.k_3 P_1^{-1} P_3^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+128 m^4 p.k_2 P_1^{-1} Q_4^{-2} [1\ 2]^{-2}$$

$$-512 m^4 p.k_2 P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$+192 m^4 p.k_1 P_1^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [1\ 4]^{-1}$$

$$+128 m^4 p.k_3 P_2^{-1} Q_4^{-2} [1\ 2]^{-1} [2\ 3]^{-1}$$

TABLE IV (CONTINUED)

$$-384 m^4 p.k_4 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$-192 m^4 p.k_2 P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-32 m^4 p.q P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-64 m^4 p.k_1 P_2^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$+480 m^4 p.k_2 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+224 m^4 p.q P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+496 m^4 p.k_2 P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-256 m^4 p.k_1 P_3^{-1} Q_4^{-2} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-64 m^4 p.q P_1^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [1\ 4]^{-1}$$

$$+128 m^4 p.k_2 P_1^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-2}$$

$$-128 m^4 p.k_2 P_1^{-1} Q_4^{-2} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-128 m^4 p.q P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$+480 m^4 p.k_4 P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$+64 m^4 p.q P_1^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [1\ 3]^{-1}$$

$$+256 m^4 p.q P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

TABLE IV (CONTINUED)

$$-128 m^4 p.k_3 P_2^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$-384 m^4 p.k_3 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-16 m^4 p.k_1 P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-704 m^4 p.q P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-704 m^4 q.k_3 P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$+64 m^4 q.k_1 P_1^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [1\ 4]^{-1}$$

$$-448 m^4 q.k_3 P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$+64 m^4 q.k_3 P_2^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$+48 m^4 q.k_3 P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+128 m^4 q.k_1 P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-640 m^4 q.k_3 P_1^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [1\ 4]^{-1}$$

$$-64 m^4 q.k_2 P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$+512 m^4 q.k_1 P_1^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [1\ 3]^{-1}$$

$$+608 m^4 q.k_3 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+64 m^4 k_1.k_4 P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& -64 m^4 k_1 k_4 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +112 m^4 k_1 k_3 P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -256 m^4 p.q p.k_2 P_1^{-1} P_3^{-1} Q_4^{-2} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -256 m^4 (p.k_2)^2 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& -64 m^4 p.q p.k_2 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& -512 m^4 (p.q)^2 P_1^{-1} P_2^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +448 m^4 p.q p.k_4 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -96 m^4 p.k_2 p.k_4 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -672 m^4 p.q p.k_4 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -288 m^4 (p.q)^2 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -80 m^4 p.k_2 p.k_4 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +128 m^4 p.k_2 p.k_4 P_1^{-1} P_3^{-1} Q_4^{-2} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -256 m^4 (p.q)^2 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& -256 m^4 (p.q)^2 P_1^{-2} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [1\ 4]^{-1} \\
& -128 m^4 (p.k_2)^2 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& -320 m^4 (p.q)^2 p_1^{-1} p_3^{-1} q_1^{-1} q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& +32 m^4 (p.k_4)^2 p_1^{-1} p_3^{-1} q_1^{-1} q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +288 m^4 (p.q)^2 p_1^{-1} p_3^{-1} q_1^{-1} q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +64 m^4 (p.k_2)^2 p_1^{-1} p_3^{-1} q_2^{-1} q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +48 m^4 (p.q)^2 p_1^{-1} p_3^{-1} q_2^{-1} q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -448 m^4 p.k_2 q.k_3 p_1^{-1} p_4^{-1} q_1^{-1} q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& -64 m^4 p.k_2 q.k_2 p_1^{-1} p_4^{-1} q_1^{-1} q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +512 m^4 p.q q.k_1 p_1^{-2} q_1^{-1} q_4^{-1} [1\ 2]^{-1} [1\ 4]^{-1} \\
& +512 m^4 p.q q.k_2 p_1^{-1} p_2^{-1} q_1^{-1} q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +192 m^4 p.k_3 q.k_1 p_1^{-1} p_2^{-1} q_3^{-1} q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& -256 m^4 p.q q.k_1 p_1^{-1} p_2^{-1} q_3^{-1} q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& -384 m^4 p.k_2 q.k_1 p_1^{-1} p_3^{-1} q_2^{-1} q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -64 m^4 p.q q.k_3 p_1^{-1} p_4^{-1} q_1^{-1} q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& -64 m^4 p.q q.k_3 p_1^{-2} q_2^{-1} q_4^{-1} [1\ 2]^{-1} [1\ 4]^{-1} \\
& -128 m^4 p.q q.k_3 p_1^{-1} p_2^{-1} q_1^{-1} q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$-128 m^4 p \cdot q \, q \cdot k_3 \, P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1 \, 2]^{-1} [2 \, 3]^{-1}$$

$$+64 m^4 p \cdot k_4 \, q \cdot k_1 \, P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [1 \, 2]^{-1} [2 \, 4]^{-1}$$

$$-320 m^4 p \cdot q \, q \cdot k_2 \, P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1 \, 2]^{-1} [3 \, 4]^{-1}$$

$$-256 m^4 (q \cdot k_3)^2 \, P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1 \, 2]^{-1} [2 \, 4]^{-1}$$

$$-64 m^4 (q \cdot k_3)^2 \, P_1^{-2} Q_2^{-1} Q_4^{-1} [1 \, 2]^{-1} [1 \, 4]^{-1}$$

$$-256 m^4 (q \cdot k_1)^2 \, P_1^{-2} Q_2^{-1} Q_4^{-1} [1 \, 2]^{-1} [1 \, 4]^{-1}$$

$$+192 m^4 q \cdot k_1 \, q \cdot k_2 \, P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [1 \, 2]^{-1} [2 \, 4]^{-1}$$

$$-128 m^4 (q \cdot k_1)^2 \, P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1 \, 2]^{-1} [3 \, 4]^{-1}$$

$$-64 m^4 q \cdot k_2 \, q \cdot k_3 \, P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1 \, 2]^{-1} [2 \, 4]^{-1}$$

$$+64 m^4 q \cdot k_1 \, q \cdot k_3 \, P_1^{-2} Q_2^{-1} Q_4^{-1} [1 \, 2]^{-1} [1 \, 4]^{-1}$$

$$-128 m^4 (q \cdot k_3)^2 \, P_1^{-1} P_2^{-1} Q_1^{-1} Q_4^{-1} [1 \, 2]^{-1} [2 \, 4]^{-1}$$

$$+64 m^4 (q \cdot k_3)^2 \, P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1 \, 2]^{-1} [3 \, 4]^{-1}$$

$$-96 m^4 q \cdot k_1 \, q \cdot k_3 \, P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1 \, 2]^{-1} [3 \, 4]^{-1}$$

$$+128 m^4 p \cdot k_2 \, k_1 \cdot k_4 \, P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1 \, 2]^{-1} [3 \, 4]^{-1}$$

$$-64 m^4 q \cdot k_2 \, k_1 \cdot k_4 \, P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1 \, 2]^{-1} [3 \, 4]^{-1}$$

TABLE IV (CONTINUED)

$$-64 m^4 q \cdot k_3 k_1 \cdot k_3 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+64 m^4 p \cdot q k_1 \cdot k_4 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-32 m^4 (k_1 \cdot k_4)^2 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+64 m^4 p \cdot q k_1 \cdot k_4 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-64 m^4 k_1 \cdot k_3 k_2 \cdot k_4 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+48 m^4 p \cdot k_2 k_1 \cdot k_3 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+48 m^4 q \cdot k_1 k_1 \cdot k_3 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+128 m^4 p \cdot q k_2 \cdot k_3 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-256 m^6 P_1^{-2} Q_4^{-2} [1\ 2]^{-1}$$

$$+256 m^6 P_1^{-1} P_3^{-1} Q_4^{-2} [1\ 2]^{-1}$$

$$+384 m^6 P_1^{-2} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1}$$

$$+384 m^6 P_1^{-1} P_2^{-1} Q_1^{-1} Q_4^{-1} [2\ 4]^{-1}$$

$$+384 m^6 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [2\ 3]^{-1}$$

$$+192 m^6 P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-1}$$

$$+64 m^6 P_1^{-1} P_2^{-1} Q_4^{-2} [1\ 2]^{-1}$$

TABLE IV (CONTINUED)

$$\begin{aligned}
& +384 m^6 p_1^{-1} p_4^{-1} q_1^{-1} q_4^{-1} [1\ 2]^{-1} \\
& +384 m^6 p_1^{-1} p_2^{-1} q_1^{-1} q_4^{-1} [1\ 2]^{-1} \\
& -224 m^6 p_1^{-1} p_3^{-1} q_1^{-1} q_4^{-1} [1\ 2]^{-1} \\
& +256 m^6 p_1^{-1} p_3^{-1} q_2^{-1} q_4^{-1} [1\ 2]^{-1} \\
& -256 m^6 p_1^{-1} q_4^{-2} [1\ 2]^{-2} \\
& -128 m^6 p_1^{-1} p_2^{-1} q_4^{-1} [1\ 2]^{-2} \\
& -192 m^6 p_1^{-1} q_2^{-1} q_4^{-1} [1\ 2]^{-1} [1\ 4]^{-1} \\
& -256 m^6 p_1^{-1} q_3^{-1} q_4^{-1} [1\ 2]^{-2} \\
& -256 m^6 p_1^{-1} q_4^{-2} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -128 m^6 p.k_3 p_1^{-1} p_2^{-1} q_4^{-2} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -128 m^6 p.q p_1^{-1} p_2^{-1} q_4^{-2} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -384 m^6 q.k_3 p_1^{-1} p_2^{-1} q_1^{-1} q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1} \\
& -256 m^6 p_3^{-1} q_1^{-1} q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& -256 m^6 p_1^{-2} q_4^{-1} [1\ 2]^{-2} \\
& -128 m^6 p_1^{-1} q_4^{-2} [1\ 2]^{-1} [1\ 3]^{-1}
\end{aligned}$$

TABLE IV (CONTINUED)

$$-192 m^6 q.k_3 P_1^{-2} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [1\ 4]^{-1}$$

$$-192 m^6 P_1^{-1} P_2^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$+128 m^6 p.k_4 P_1^{-1} P_2^{-1} Q_4^{-2} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-384 m^6 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$-384 m^6 p.k_2 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$+64 m^6 p.q P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-64 m^6 p.k_2 P_1^{-1} P_3^{-1} Q_4^{-2} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-192 m^6 p.q P_1^{-1} P_3^{-1} Q_4^{-2} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$+192 m^6 p.k_4 P_1^{-1} P_3^{-1} Q_4^{-2} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-192 m^6 p.k_2 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$-192 m^6 q.k_3 P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$+256 m^6 q.k_1 P_1^{-2} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [1\ 3]^{-1}$$

$$+64 m^6 P_2^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-256 m^6 p.q P_1^{-1} P_2^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$+64 m^6 p.k_3 P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

TABLE IV (CONTINUED)

$$+192 m^6 q.k_2 P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$+352 m^6 P_1^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-128 m^6 q.k_3 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-128 m^6 p.q P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+128 m^6 q.k_1 P_1^{-1} P_2^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-2}$$

$$+64 m^6 k_2.k_4 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-384 m^6 p.q P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-256 m^6 p.q P_1^{-2} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [1\ 3]^{-1}$$

$$-256 m^6 p.q P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$-192 m^6 p.k_4 P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$-32 m^6 P_1^{-1} P_2^{-1} Q_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$-32 m^6 p.k_4 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+64 m^6 k_1.k_4 P_1^{-1} P_3^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-64 m^6 p.q P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-2}$$

$$+160 m^6 p.q P_1^{-1} P_4^{-1} Q_1^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

TABLE IV (CONTINUED)

$$+592 m^6 P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-128 m^8 P_1^{-2} Q_4^{-2} [1\ 2]^{-2}$$

$$+64 m^8 P_1^{-1} P_3^{-1} Q_4^{-2} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-128 m^8 P_1^{-2} Q_3^{-1} Q_4^{-1} [1\ 2]^{-2}$$

$$-64 m^8 P_1^{-1} P_2^{-1} Q_3^{-1} Q_4^{-1} [1\ 2]^{-2}$$

$$-128 m^8 P_1^{-1} P_2^{-1} Q_4^{-2} [1\ 2]^{-2}$$

$$-128 m^8 P_1^{-2} Q_4^{-2} [1\ 2]^{-1} [1\ 3]^{-1}$$

$$-128 m^8 P_1^{-1} P_2^{-1} Q_4^{-2} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$+176 m^8 P_1^{-1} P_3^{-1} Q_2^{-1} Q_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \}$$

TABLE V

NON-RELATIVISTIC LIMIT OF REDUCED CROSS SECTION: $e^+ + e^- \rightarrow 4\gamma$

$$\begin{aligned}
\sigma_r = 24 A \frac{1}{4m^2} \frac{1}{2^6} \left\{ \right. \\
& +1536 m^{-2} \omega_2^{-1} \omega_4^{-1} \\
& +512 m^{-2} \omega_3 \omega_1^{-1} \omega_2^{-1} \omega_4^{-1} \\
& +64 \omega_1^{-2} \omega_4^{-2} \\
& -608 \omega_1^{-1} \omega_2^{-1} \omega_3^{-1} \omega_4^{-1} \\
& +192 m^{-1} \omega_1^{-1} \omega_2^{-1} \omega_4^{-1} \\
& +128 m^{-1} \omega_1^{-2} \omega_4^{-1} \\
& -320 m^{-1} \omega_3 \omega_1^{-2} \omega_2^{-1} \omega_4^{-1} \\
& +256 m^{-1} \omega_2 \omega_1^{-2} \omega_4^{-2} \\
& +416 m^{-2} k_2 \cdot k_4 \omega_1^{-1} \omega_2^{-1} \omega_3^{-1} \omega_4^{-1} \\
& +448 \omega_1^{-2} \omega_2^{-1} \omega_4^{-1} \\
& +256 m^{-2} k_1 \cdot k_3 \omega_1^{-2} \omega_3^{-1} \omega_4^{-1} \\
& \left. +1280 m^{-1} \omega_4 \omega_1^{-1} \omega_3^{-1} [1 \ 2]^{-1} \right\}
\end{aligned}$$

TABLE V (CONTINUED)

$$+256 \omega_1^{-2} [1 \ 2]^{-1}$$

$$-512 m^{-1} \omega_1 \omega_2^{-1} \omega_4^{-1} [2 \ 4]^{-1}$$

$$-128 m^{-1} k_1 \cdot k_3 \omega_1^{-1} \omega_2^{-1} \omega_3^{-1} [1 \ 2]^{-1}$$

$$-128 m^{-2} k_1 \cdot k_3 \omega_3^{-1} \omega_4^{-1} [3 \ 4]^{-1}$$

$$+128 m^{-2} k_1 \cdot k_3 \omega_2^{-1} \omega_4^{-1} [1 \ 2]^{-1}$$

$$-1344 m \omega_1^{-2} \omega_4^{-1} [1 \ 2]^{-1}$$

$$+736 \omega_2 \omega_1^{-2} \omega_3^{-1} [1 \ 2]^{-1}$$

$$+1120 m^{-1} \omega_1^{-1} [1 \ 2]^{-1}$$

$$-32 \omega_1^{-1} \omega_4^{-1} [1 \ 2]^{-1}$$

$$+1184 \omega_1^{-1} \omega_2^{-1} [1 \ 2]^{-1}$$

$$+128 m^{-1} k_2 \cdot k_3 \omega_1^{-1} \omega_3^{-1} \omega_4^{-1} [1 \ 2]^{-1}$$

$$+320 k_1 \cdot k_3 \omega_1^{-1} \omega_2^{-1} \omega_3^{-1} \omega_4^{-1} [1 \ 2]^{-1}$$

$$-128 m^{-2} \omega_3 k_1 \cdot k_2 \omega_1^{-1} \omega_2^{-1} \omega_4^{-1} [2 \ 3]^{-1}$$

$$+64 m^{-2} (k_1 \cdot k_3)^2 \omega_1^{-1} \omega_2^{-1} \omega_3^{-1} \omega_4^{-1} [1 \ 2]^{-1}$$

$$+1600 \omega_3 \omega_1^{-1} \omega_2^{-1} \omega_4^{-1} [1 \ 4]^{-1}$$

TABLE V (CONTINUED)

$$\begin{aligned}
& +1088 \omega_4 \omega_1^{-2} \omega_3^{-1} [1 \ 2]^{-1} \\
& -448 m^{-1} k_2 \cdot k_4 \omega_1^{-2} \omega_4^{-1} [1 \ 2]^{-1} \\
& -256 k_1 \cdot k_4 \omega_1^{-2} \omega_4^{-2} [1 \ 2]^{-1} \\
& -192 m^{-1} k_1 \cdot k_3 \omega_1^{-2} \omega_3^{-1} [1 \ 2]^{-1} \\
& -64 m^{-1} k_1 \cdot k_3 \omega_1^{-2} \omega_4^{-1} [1 \ 2]^{-1} \\
& -64 m^{-2} k_1 \cdot k_3 k_1 \cdot k_4 \omega_1^{-2} \omega_4^{-2} [1 \ 2]^{-1} \\
& -128 m^{-1} k_1 \cdot k_2 \omega_1^{-1} \omega_4^{-2} [2 \ 3]^{-1} \\
& +128 m^{-1} \omega_2 k_2 \cdot k_3 \omega_1^{-2} \omega_3^{-1} \omega_4^{-1} [1 \ 2]^{-1} \\
& -64 m^{-1} k_2 \cdot k_3 \omega_1^{-2} \omega_4^{-1} [1 \ 2]^{-1} \\
& -64 k_2 \cdot k_3 \omega_1^{-2} \omega_3^{-1} \omega_4^{-1} [1 \ 2]^{-1} \\
& +320 k_1 \cdot k_4 \omega_1^{-2} \omega_3^{-1} \omega_4^{-1} [1 \ 2]^{-1} \\
& +64 m^{-1} \omega_2 k_1 \cdot k_4 \omega_1^{-2} \omega_4^{-2} [1 \ 2]^{-1} \\
& +256 m^{-1} k_1 \cdot k_2 \omega_1^{-2} \omega_4^{-1} [2 \ 3]^{-1} \\
& -128 m^{-1} \omega_3 k_2 \cdot k_4 \omega_1^{-2} \omega_4^{-2} [1 \ 2]^{-1} \\
& +192 m^{-1} \omega_2 k_1 \cdot k_2 \omega_1^{-2} \omega_3^{-1} \omega_4^{-1} [2 \ 3]^{-1}
\end{aligned}$$

TABLE V (CONTINUED)

$$\begin{aligned}
& -384 k_1 \cdot k_3 \omega_1^{-2} \omega_2^{-1} \omega_4^{-1} [1 \ 2]^{-1} \\
& -128 k_1 \cdot k_2 \omega_1^{-2} \omega_2^{-1} \omega_4^{-1} [2 \ 3]^{-1} \\
& -128 m^{-1} \omega_2 k_2 \cdot k_4 \omega_1^{-2} \omega_4^{-2} [1 \ 2]^{-1} \\
& -128 m^{-1} \omega_3 k_1 \cdot k_2 \omega_1^{-1} \omega_2^{-1} \omega_4^{-2} [2 \ 3]^{-1} \\
& +64 m \omega_3^{-1} \omega_4^{-2} [1 \ 2]^{-1} \\
& -1344 m \omega_2 \omega_1^{-1} \omega_3^{-1} \omega_4^{-2} [1 \ 2]^{-1} \\
& +384 \omega_2 \omega_3 \omega_1^{-2} \omega_4^{-2} [1 \ 2]^{-1} \\
& +1152 m \omega_4 \omega_1^{-1} \omega_2^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1} \\
& +960 m^3 \omega_1^{-2} \omega_2^{-1} [1 \ 2]^{-1} [1 \ 4]^{-1} \\
& +896 m^3 \omega_1^{-1} \omega_2^{-1} \omega_4^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1} \\
& -448 m^2 \omega_3 \omega_1^{-2} \omega_2^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1} \\
& -32 \omega_2 k_1 \cdot k_3 \omega_1^{-2} \omega_3^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& +32 \omega_2 k_1 \cdot k_4 \omega_1^{-2} \omega_3^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
& +64 m^{-1} \omega_3 k_1 \cdot k_2 \omega_1^{-2} \omega_2^{-1} \omega_4^{-1} [2 \ 3]^{-1} \\
& +64 k_2 \cdot k_4 \omega_1^{-2} \omega_4^{-2} [1 \ 2]^{-1}
\end{aligned}$$

TABLE V (CONTINUED)

$$+192 \omega_2 \omega_1^{-2} \omega_4^{-1} [2 \ 3]^{-1}$$

$$+896 m^2 \omega_1^{-2} \omega_3^{-1} \omega_4^{-1} [2 \ 3]^{-1}$$

$$-64 m \omega_2 \omega_1^{-2} \omega_3^{-1} \omega_4^{-1} [1 \ 2]^{-1}$$

$$+192 \omega_2^2 \omega_1^{-2} \omega_3^{-1} \omega_4^{-1} [2 \ 3]^{-1}$$

$$-128 m \omega_2^2 \omega_1^{-2} \omega_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1}$$

$$-32 m \omega_2 \omega_3 \omega_1^{-1} \omega_4^{-2} [1 \ 2]^{-1} [3 \ 4]^{-1}$$

$$-64Q m^2 \omega_3 \omega_1^{-2} \omega_2^{-1} [1 \ 2]^{-1} [1 \ 3]^{-1}$$

$$+64 m^{-1} k_1 \cdot k_3 k_2 \cdot k_3 \omega_1^{-2} \omega_3^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1}$$

$$+512 m k_1 \cdot k_4 \omega_1^{-1} \omega_4^{-2} [1 \ 2]^{-1} [3 \ 4]^{-1}$$

$$-3520 m \omega_1^{-1} \omega_2^{-1} \omega_4^{-1} [2 \ 4]^{-1}$$

$$-1216 m \omega_2 \omega_1^{-2} \omega_4^{-2} [1 \ 2]^{-1}$$

$$+896 m^2 \omega_1^{-1} \omega_2^{-1} \omega_3^{-1} \omega_4^{-1} [1 \ 2]^{-1}$$

$$+96 \omega_2^2 \omega_1^{-2} \omega_3^{-1} \omega_4^{-1} [1 \ 2]^{-1}$$

$$-32 [1 \ 2]^{-2}$$

$$-64 \omega_4 \omega_1^{-1} [1 \ 2]^{-1} [1 \ 3]^{-1}$$

TABLE V (CONTINUED)

$$+352 m \omega_4^{-1} [1 \ 2]^{-2}$$

$$+384 \omega_2 \omega_4 \omega_1^{-1} \omega_3^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1}$$

$$-96 m^2 \omega_1^{-1} \omega_3^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1}$$

$$-704 m \omega_3 \omega_1^{-1} \omega_4^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1}$$

$$-576 m^2 \omega_2^{-1} \omega_4^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1}$$

$$-704 m \omega_2 \omega_1^{-2} [1 \ 2]^{-1} [2 \ 3]^{-1}$$

$$+512 \omega_2^2 \omega_1^{-2} \omega_4^{-2} [1 \ 2]^{-1}$$

$$-512 m^2 \omega_1^{-2} \omega_3^{-1} \omega_4^{-1} [1 \ 2]^{-1}$$

$$+768 m^2 \omega_1^{-2} \omega_4^{-2} [1 \ 2]^{-1}$$

$$-320 [1 \ 2]^{-1} [2 \ 3]^{-1}$$

$$+128 \omega_4 \omega_3^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1}$$

$$-1536 m \omega_2^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1}$$

$$+32 \omega_4 \omega_3^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1}$$

$$+384 \omega_2^2 \omega_1^{-1} \omega_3^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1}$$

$$-320 m^2 \omega_1^{-1} \omega_3^{-1} [1 \ 2]^{-1} [2 \ 3]^{-1}$$

TABLE V (CONTINUED)

$$+384 k_1 \cdot k_4 \omega_1^{-1} \omega_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-320 m \omega_3 \omega_1^{-2} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-128 m^2 \omega_4^{-2} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$+800 m^2 \omega_3^{-1} \omega_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+2144 m^2 \omega_1^{-2} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-672 m \omega_2 \omega_1^{-2} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-128 m^{-2} k_1 \cdot k_3 k_2 \cdot k_3 \omega_2^{-1} \omega_3^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-64 m^{-1} \omega_4 k_1 \cdot k_3 \omega_2^{-1} \omega_3^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-1088 m^2 \omega_2 \omega_1^{-1} \omega_4^{-2} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$+1856 m^3 \omega_1^{-2} \omega_3^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$+64 m \omega_2^2 \omega_1^{-2} \omega_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$-2240 m^3 \omega_1^{-2} \omega_3^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-64 m \omega_2 \omega_4 \omega_1^{-2} \omega_3^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-384 m^{-1} k_2 \cdot k_3 \omega_1^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+672 m \omega_2 \omega_1^{-1} \omega_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

TABLE V (CONTINUED)

$$\begin{aligned}
& -1024 m \omega_4 \omega_1^{-2} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -256 \omega_1 \omega_4 \omega_2^{-1} \omega_3^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +64 k_2 \cdot k_3 \omega_1^{-1} \omega_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +1216 m \omega_2^2 \omega_1^{-1} \omega_3^{-1} \omega_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& +672 m^2 \omega_2 \omega_1^{-1} \omega_4^{-2} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -2816 m^2 \omega_2 \omega_1^{-1} \omega_3^{-1} \omega_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& +544 m^2 \omega_3 \omega_1^{-1} \omega_4^{-2} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -448 m \omega_2 \omega_3^{-1} \omega_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +896 k_2 \cdot k_3 \omega_1^{-1} \omega_3^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -384 m k_2 \cdot k_3 \omega_1^{-1} \omega_3^{-1} \omega_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +64 m^{-2} (k_1 \cdot k_3)^2 \omega_1^{-1} \omega_3^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -320 m^{-1} k_1 \cdot k_3 \omega_3^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -768 m k_1 \cdot k_3 \omega_1^{-1} \omega_3^{-1} \omega_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -1632 m^3 \omega_1^{-1} \omega_3^{-1} \omega_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +64 m^{-2} k_1 \cdot k_3 k_2 \cdot k_4 \omega_2^{-1} \omega_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}
\end{aligned}$$

TABLE V (CONTINUED)

$$+64 m^{-1} \omega_1 k_1 \cdot k_3 \omega_2^{-1} \omega_3^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+1024 m^3 \omega_2 \omega_1^{-2} \omega_4^{-2} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$+1088 m^4 \omega_1^{-2} \omega_3^{-1} \omega_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-320 m^4 \omega_1^{-2} \omega_4^{-2} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+448 m^4 \omega_1^{-1} \omega_2^{-1} \omega_3^{-1} \omega_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$+96 m^2 \omega_1 \omega_2^{-1} \omega_3^{-1} \omega_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-32 \omega_1 k_1 \cdot k_3 \omega_2^{-1} \omega_3^{-1} \omega_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-192 m^3 \omega_2 \omega_1^{-1} \omega_3^{-1} \omega_4^{-2} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$+192 m \omega_2^3 \omega_1^{-2} \omega_4^{-2} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$-512 m^2 \omega_2^2 \omega_1^{-2} \omega_4^{-2} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$-320 m^2 \omega_2^2 \omega_1^{-2} \omega_3^{-1} \omega_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1}$$

$$-896 m^4 \omega_1^{-2} \omega_2^{-1} \omega_4^{-1} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$+448 m^3 \omega_2 \omega_1^{-2} \omega_4^{-2} [1\ 2]^{-1} [3\ 4]^{-1}$$

$$-448 m^2 \omega_2 \omega_3 \omega_1^{-2} \omega_4^{-2} [1\ 2]^{-1} [2\ 4]^{-1}$$

$$-64 m^{-1} (k_1 \cdot k_4)^2 \omega_1^{-2} \omega_3^{-1} [1\ 2]^{-1} [3\ 4]^{-1}$$

TABLE V (CONTINUED)

$$\begin{aligned}
& -64 m^{-1} k_1 \cdot k_4 k_2 \cdot k_3 \omega_1^{-2} \omega_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +64 m^{-1} \omega_2 (k_1 \cdot k_4)^2 \omega_1^{-2} \omega_4^{-2} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -32 k_1 \cdot k_3 \omega_3^{-1} \omega_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -512 m^4 \omega_1^{-2} \omega_4^{-2} [1\ 2]^{-1} [2\ 4]^{-1} \\
& +128 m^4 \omega_1^{-2} \omega_2^{-1} \omega_4^{-1} [1\ 2]^{-1} [1\ 4]^{-1} \\
& +64 m^2 \omega_2^2 \omega_1^{-2} \omega_4^{-2} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +576 m^3 \omega_2 \omega_1^{-2} \omega_3^{-1} \omega_4^{-1} [1\ 2]^{-1} [2\ 3]^{-1} \\
& +448 m \omega_2^2 \omega_3 \omega_1^{-2} \omega_4^{-2} [1\ 2]^{-1} [2\ 4]^{-1} \\
& -288 m^3 \omega_2 \omega_1^{-2} \omega_3^{-1} \omega_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +128 k_1 \cdot k_3 k_2 \cdot k_4 \omega_1^{-2} \omega_3^{-1} \omega_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -64 m \omega_2 k_1 \cdot k_3 \omega_1^{-2} \omega_3^{-1} \omega_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +128 m^2 k_1 \cdot k_3 \omega_1^{-2} \omega_3^{-1} \omega_4^{-1} [1\ 2]^{-1} [3\ 4]^{-1} \\
& -128 m^2 k_1 \cdot k_4 \omega_1^{-2} \omega_4^{-2} [1\ 2]^{-1} [3\ 4]^{-1} \\
& +256 k_1 \cdot k_2 \omega_1^{-1} \omega_2^{-1} \omega_4^{-2} [2\ 3]^{-1} \\
& -64 m^2 \omega_3^2 \omega_1^{-2} \omega_2^{-1} \omega_4^{-1} [1\ 2]^{-1} [1\ 4]^{-1}
\end{aligned}$$

TABLE V (CONTINUED)

$$\begin{aligned}
 & -64 k_1 \cdot k_3 k_2 \cdot k_4 \omega_1^{-1} \omega_2^{-1} \omega_3^{-1} \omega_4^{-1} [1 \ 2]^{-1} [3 \ 4]^{-1} \\
 & +256 m^4 \omega_1^{-1} \omega_2^{-1} \omega_3^{-1} \omega_4^{-1} [1 \ 2]^{-1} [2 \ 4]^{-1} \}
 \end{aligned}$$

BIBLIOGRAPHY

- Andreassi, G., Calucci, G., Furlan, G., Peressutti, G., and Cazzola, P., "Radiative Corrections to the Total Cross Section for Annihilation of a Pair into Photons," Phys. Rev., Vol. 128 (1962), p. 1425.
- Braccini, P. L., Ion, I. X., Stefanini, A., Torelli, G., and Torelli Tosi, R., "Annihilation in Flight of 800 Mev Positrons," Nuovo Cimento, Vol. 29 (1963), p. 1215.
- Brown, Laurie M., "Two Component Fermion Theory," Phys. Rev., Vol. 111 (1958), p. 957.
- Budini, P., and Furlan, G., "High Energy Behavior of Electrodynamical Cross Sections," Nuovo Cimento, Vol. 29 (1963), p. 451.
- Calogero, F., and Zemach, C., "Particle Creation in Electron-Electron Collisions," Phys. Rev., Vol. 120 (1960), p. 1860.
- DeBenedetti, A., Garelli, C. M., Tallone, L., Vigone, M., and Wataghin, G., "On Narrow Showers of Pairs of Charged Particles," Nuovo Cimento, Vol. 12 (1954), p. 954.
- Dirac, P. A. M., "Annihilation of Electrons and Protons," Proc. Cambridge Phil. Soc., Vol. 26 (1930), p. 361.
- Fabiani, F., Fidecaro, M., Finocchiaro, G., Giacomelli, G., Harting, D., Lipman, N. H., and Torelli, G., "A Measurement of the Total Cross Section for the Annihilation in Flight of Positrons Between 2 and 10 Gev," The Aix-En-Provence International Conference on Elementary Particles, Vol. 1, (ed. by E. Cremieu-Alcan, P. Falk-Vairant, and O. Lebey), p. 151, Saclay, France, 1962.
- Gupta, Suraj N., "Multiple Photon Production in Quantum Electrodynamics," Phys. Rev., Vol. 98 (1955), p. 1502.

- Hearn, A. C., "Symbolic Computation of Feynman Graphs, Using a Digital Computer," Bull. Am. Phys. Soc., Vol. 9 (1964), p. 436.
- Jauch, J. M., and Rohrlich, F., The Theory of Photons and Electrons, pp. 144, 167, 437, Reading, Massachusetts, 1955.
- Joseph, James, "Multiple Photon Production by Electron Pair Annihilation in Flight," Phys. Rev., Vol. 103 (1956), p. 481.
- Mandl, F., and Skyrme, T. H. R., "The Theory of the Double Compton Effect," Proc. Roy. Soc. A., Vol. 215 (1952), p. 497.
- Rose, M. E., Relativistic Electron Theory, pp. 9, 246, New York, 1961.
- Sannikov, S. S., "High Energy Electron Scattering Processes," Soviet Phys. - JETP, Vol. 13 (1961), p. 163.
- Schein, Marcel, Haskin, D. M., and Glasser, R. G., "Narrow Shower of Pure Photons at 100,000 Feet," Phys. Rev., Vol. 95 (1954), p. 855.
- Wilcox, Ralph Myron, A Computer Method for Calculating Dirac Traces With Applications to Quantum Electrodynamics, (unpublished doctoral dissertation, University of Colorado, 1961).

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Abstract of
FOUR PHOTON ANNIHILATION
OF AN ELECTRON PAIR

A method for reducing the labor involved in quantum electrodynamical calculations is developed and applied to the process of four photon annihilation of an electron positron pair. It utilizes the concept of a reduced cross section, an abbreviated form of the differential cross section which contains all the information necessary to determine any differential or partial cross section. The reduced cross section is obtained from the differential cross section by effecting an identity between all terms which differ only by a permutation of identical final state particles. This technique has been used elsewhere but only for the total cross section. This thesis proves that it can be applied to any partial cross section.

The reduced cross section for four photon annihilation of an electron positron pair is calculated for arbitrary energy. It is integrated for the special case in the nonrelativistic limit in which two of the photons are soft

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and one hard photon is emitted in a small cone centered on the direction of the incident positron. This result is used to estimate the total cross section for in-flight annihilation in the nonrelativistic limit, $\sigma \approx \frac{2^4}{3\pi}$

$\frac{r_0^2 \alpha^2}{\beta}$ (1.2×10^{-2}), and the lifetime of positronium for four photon decay from the 1^1S_0 state, $\tau \approx 3.6 \times 10^{-4}$ sec.

The concept of the reduced cross section affords a considerable economy in computational labor. The four photon annihilation calculation would probably not have been feasible otherwise. However an estimate of the work required to compute the five photon annihilation process indicates that even a calculation of the reduced cross section for this process is not feasible. A more radical simplification is required before these higher order processes can be computed. There remain, however, a number of processes with three and four corner diagrams to which the concept of the reduced cross section can be profitably applied.